



CP violation and the Heavy Flavour

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Outline

- Symmetries : C, P, CP, T and CPT
- Flavour Mixing
- SM CP violation : KM Mechanism
- Heavy Flavour and CP violation
- Needs for NP !!

Parity & Charge Conjugation

What is CP?

- What is P? **Answer:** parity ($\vec{r} \rightarrow -\vec{r}$ & $\vec{p} \rightarrow -\vec{p}$)

$$P|\Psi(\vec{r}, s_z)\rangle = \pm|\Psi(-\vec{r}, s_z)\rangle$$

- What is C? **Answer:** charge conjugation.

$$C|e^-\rangle = |e^+\rangle \quad C|p\rangle = |\bar{p}\rangle \quad C|\gamma\rangle = -|\gamma\rangle$$

Note that C also flips lepton and baryon number. Note further that neutral particles can be eigenstates of C.

Parity Falls

Until the mid-50's, people believed that both P and C would be conserved. However, in 1957, Wu et al., who were pursuing ideas of Lee & Yang (inspired by experimental data on K decays) observed parity violation in nuclear β decay. Although parity is conserved in strong and electromagnetic interactions it is in a sense “maximally violated” in weak interactions.

Parity conservation in QED and QCD

- Consider the QED process $e^- q \rightarrow e^- q$

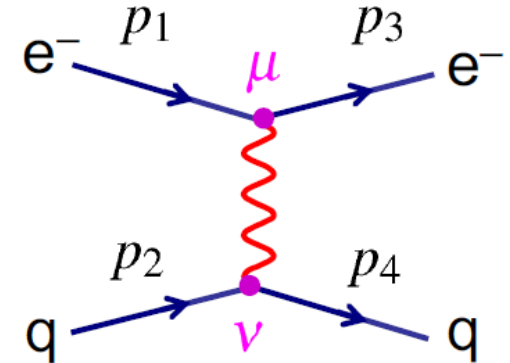
- The Feynman rules for QED give:

$$-iM = [\bar{u}_e(p_3) i e \gamma^\mu u_e(p_1)] \frac{-i g_{\mu\nu}}{q^2} [\bar{u}_q(p_4) i e \gamma^\nu u_q(p_2)]$$

- Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2} g_{\mu\nu} j_e^\mu j_q^\nu = -\frac{e^2}{q^2} j_e \cdot j_q$$

with $j_e = \bar{u}_e(p_3) \gamma^\mu u_e(p_1)$ and $j_q = \bar{u}_q(p_4) \gamma^\mu u_q(p_2)$



- ★ Consider the what happen to the matrix element under the parity transformation

- ♦ Spinors transform as

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$$

- ♦ Adjoint spinors transform as

$$\bar{u} = u^\dagger \gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger \gamma^0 = u^\dagger \gamma^{0\dagger} \gamma^0 = u^\dagger \gamma^0 \gamma^0 = \bar{u} \gamma^0$$

$$\bar{u} \xrightarrow{\hat{P}} \bar{u} \gamma^0$$

- ♦ Hence $j_e = \bar{u}_e(p_3) \gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \bar{u}_e(p_3) \gamma^0 \gamma^\mu \gamma^0 u_e(p_1)$

★ Consider the components of the four-vector current

0: $j_e^0 \xrightarrow{\hat{P}} \bar{u} \gamma^0 \gamma^0 \gamma^0 u = \bar{u} \gamma^0 u = j_e^0$ since $\gamma^0 \gamma^0 = 1$

k=1,2,3: $j_e^k \xrightarrow{\hat{P}} \bar{u} \gamma^0 \gamma^k \gamma^0 u = -\bar{u} \gamma^k \gamma^0 \gamma^0 u = -\bar{u} \gamma^k u = -j_e^k$ since $\gamma^0 \gamma^k = -\gamma^k \gamma^0$

- The time-like component remains unchanged and the space-like components change sign

• Similarly $j_q^0 \xrightarrow{\hat{P}} j_q^0 \quad j_q^k \xrightarrow{\hat{P}} -j_q^k \quad k = 1, 2, 3$

★ Consequently the four-vector scalar product

$$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q \quad k = 1, 2, 3$$

or $j^\mu \xrightarrow{\hat{P}} j_\mu$
 $j^\mu \cdot j^\nu \xrightarrow{\hat{P}} j_\mu \cdot j_\nu$
 $\xrightarrow{\hat{P}} j^\mu \cdot j^\nu$

QED Matrix Elements are Parity Invariant



Parity Conserved in QED


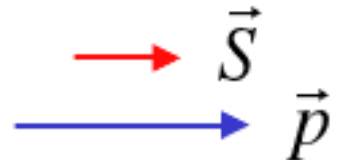
★ The QCD vertex has the same form and, similarly,

Parity Conserved in QCD

Parity changes LH to RH

Parity Falls

In particular, neutrinos, which are massless (or nearly so), have a definite “handedness”.

- $V'S$ are **left handed**

- $\bar{V}'S$ are **right handed**.


In a “symmetric” interaction, one would expect both helicities to exist, as is the case, for example, in electromagnetism, where photons have both left- and right-circular polarizations.

Parity violation: $\tau - \theta$ puzzle

The downfall of parity began in 1950's

✓ Two identical particles τ and θ

same mass, spin , charge etc...

✓ $\theta^+ \rightarrow \pi^+ \pi^0$ (P=+1)
 $\tau^+ \rightarrow \left\{ \begin{array}{l} \pi^+ \pi^+ \pi^- \\ \pi^+ \pi^0 \pi^0 \end{array} \right\}$ (P=-1) > The products in the two reactions have opposite parity !!

✓ Lee and Yang proposed that parity may not be conserved in weak interactions !!

Our lives are predominantly governed by the electromagnetic and gravitational interactions, forces of nature which *do* exhibit mirror symmetry, and so it makes sense that parity conservation is intuitive to us, but evidently this does not hold true in the weak case.

Hard to believe at that point of time ,
further experimental evidences !!

Parity violation in β -decay

★ The parity operator \hat{P} corresponds to a discrete transformation $x \rightarrow -x$, etc.

★ Under the parity transformation:

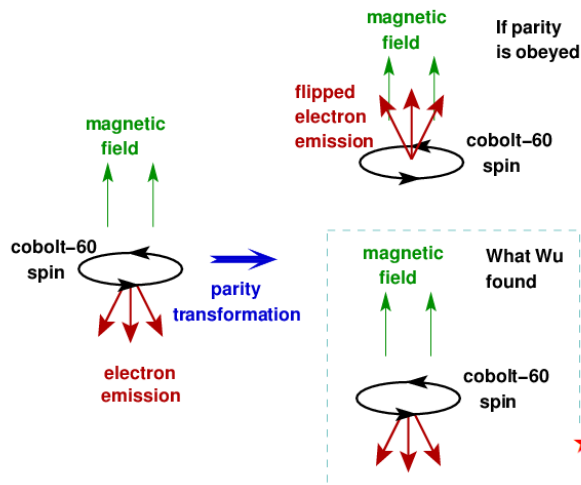
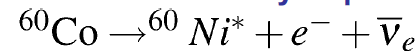
$$\begin{array}{ll} \text{Vectors} & \left\{ \begin{array}{l} \vec{r} \xrightarrow{\hat{P}} -\vec{r} \\ \vec{p} \xrightarrow{\hat{P}} -\vec{p} \end{array} \right. \\ \text{change sign} & (p_x = \frac{\partial}{\partial x}, \text{ etc.}) \\ \text{Axial-Vectors} & \left\{ \begin{array}{l} \vec{L} \xrightarrow{\hat{P}} \vec{L} \\ \vec{\mu} \xrightarrow{\hat{P}} \vec{\mu} \end{array} \right. \\ \text{unchanged} & (\vec{L} = \vec{r} \wedge \vec{p}) \\ & (\vec{\mu} \propto \vec{L}) \end{array}$$

Note B is an axial vector
 $d\vec{B} \propto \vec{J} \wedge \vec{r} d^3\vec{r}$

✓ Some nuclei have non-zero spin and can be polarized by placing in a magnetic field

✓ Beta decay of a nucleus can be used as a test of parity conservation when the magnetic moment of the nucleus is polarized in the z-direction.

★ 1957: C.S.Wu et al. studied beta decay of polarized cobalt-60 nuclei:



If parity were conserved:
expect equal rate for
producing e^- in directions
along and opposite to the
nuclear spin.

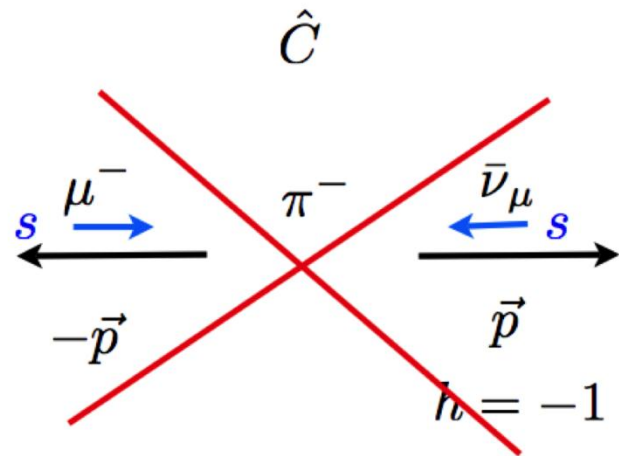
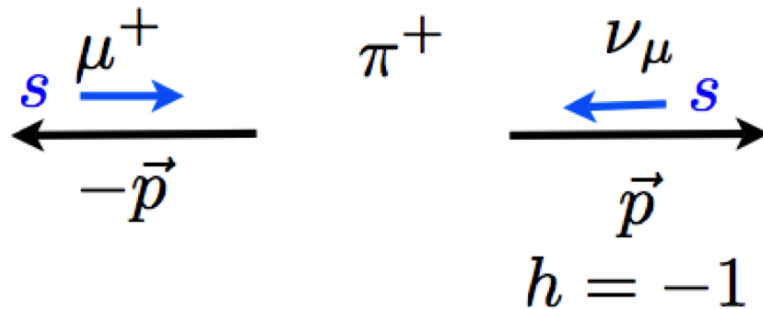
Pairs of left handed
electrons and right
handed anti-neutrinos
are emitted !!
No left handed anti-
neutrinos !!

★ Conclude parity is violated in WEAK INTERACTION

→ that the WEAK interaction vertex is NOT of the form $\bar{u}_e \gamma^\mu u_\nu$

C-violation in weak interactions !!

- Wu's experiment was shortly followed by another clever experiment by L. Lederman: Look at decay $\pi^+ \rightarrow \mu^+ \nu_\mu$
 - Pion has spin 0, μ, ν_μ both have spin $1/2$
 - \rightarrow spin of decay products must be oppositely aligned
 - \rightarrow Helicity of muon is same as that of neutrino.



- Nice feature: can also measure polarization of both neutrino (π^+ decay) and anti-neutrino (π^- decay)
- Lederman's result: All **neutrinos are left-handed** and all **anti-neutrinos are right-handed**

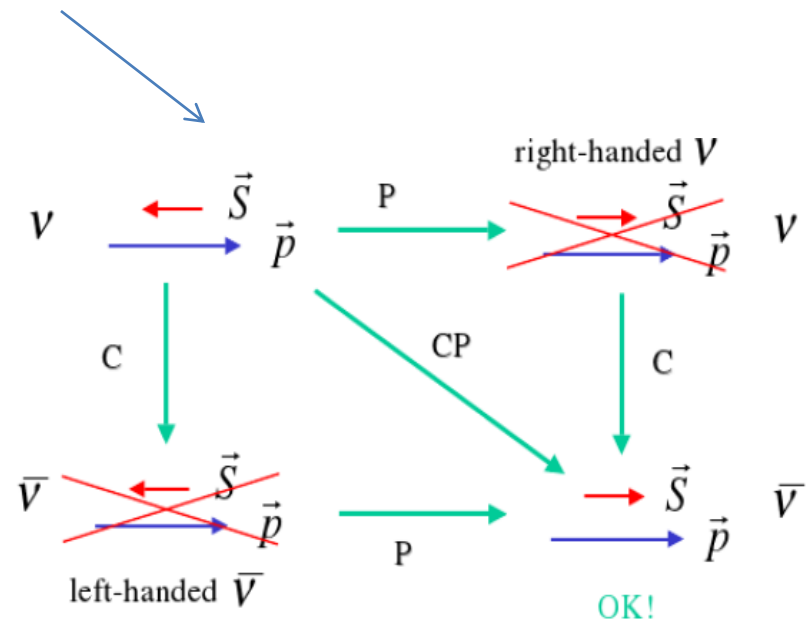
CP conservation !!

- CP is a discrete symmetry of nature given by the product of two components: charge conjugation (C) and parity (P).
- Therefore, when we apply a CP transformation to an electron moving with a velocity \mathbf{v} we will obtain a positron moving with a velocity $-\mathbf{v}$. This means that applying CP on matter gives us the mirror image of the corresponding anti-matter.

Both electromagnetic and strong interactions are symmetric under C and P, therefore they must also be symmetric under the product CP.

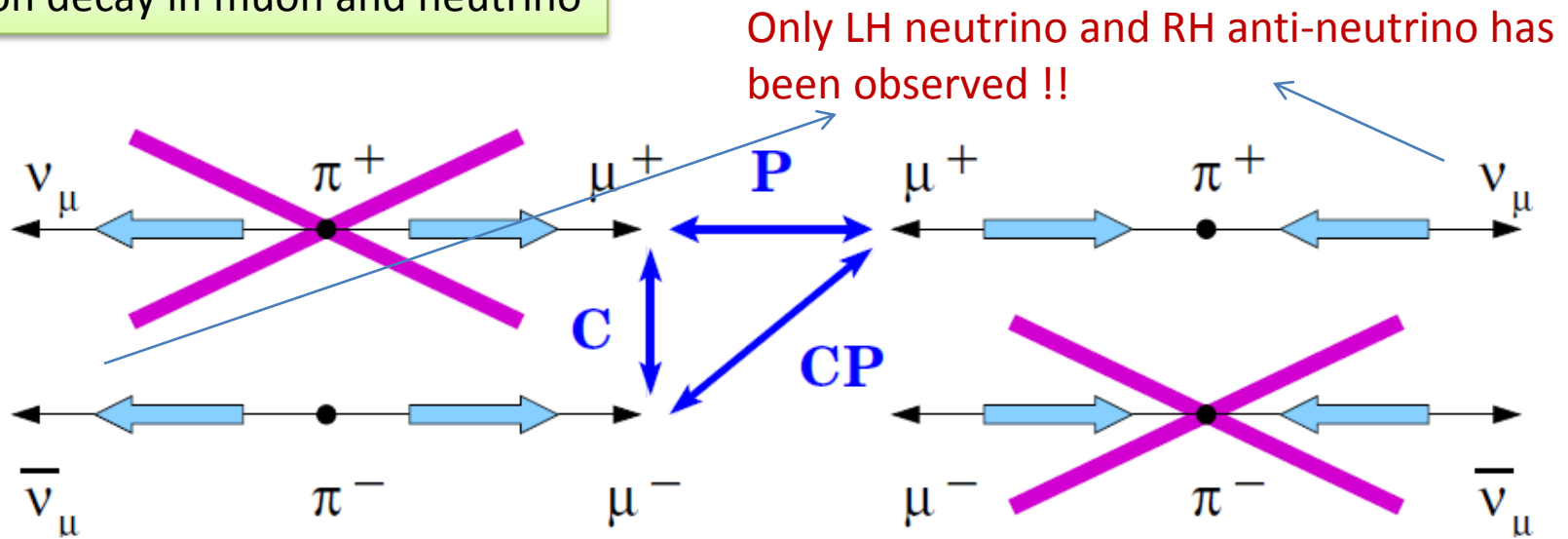


Not necessarily the case for the weak force, which violates both C and P Symmetries !!



CP parity

Pion decay in muon and neutrino



Simultaneous operation of C and P, the so called CP, recovers the symmetry !!

The CP symmetry was then assumed to be conserved in all kinds of interactions !!

CP violation

Evidence for CP violation in the decay of neutral K-mesons
observed by James Cronin & Val Fitch in 1964

- ▶ **strong eigenstates:** $|K^0\rangle = |d\bar{s}\rangle$ & $|\bar{K}^0\rangle = |s\bar{d}\rangle$
- ▶ mixing of K^0 & \bar{K}^0 via weak interaction
- ▶ physical states are superposition of K^0 & \bar{K}^0
- ▶ K^0 & \bar{K}^0 are no eigenstates of CP: $CP |K^0\rangle = -|\bar{K}^0\rangle$
 $CP |\bar{K}^0\rangle = -|K^0\rangle$
- ▶ **CP eigenstates** are linear combinations of the strong eigenstates:

$$|K_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle), \quad CP |K_1\rangle = +|K_1\rangle \quad \text{"CP even"}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle), \quad CP |K_2\rangle = -|K_2\rangle \quad \text{"CP odd"}$$

CP-violation in kaon

- ▶ for the final states of neutral kaon **decays to pions** one finds:

e.g. for $K^0 \rightarrow \pi^0 \pi^0$ $J^P : 0^- \rightarrow 0^- 0^- \Rightarrow L = 0$

$$CP |\pi^0 \pi^0\rangle = P |\pi^0 \pi^0\rangle = (-1)^2 (-1)^L = + |\pi^0 \pi^0\rangle$$

$$CP |\pi\pi\rangle = + |\pi\pi\rangle \quad \text{CP even}$$

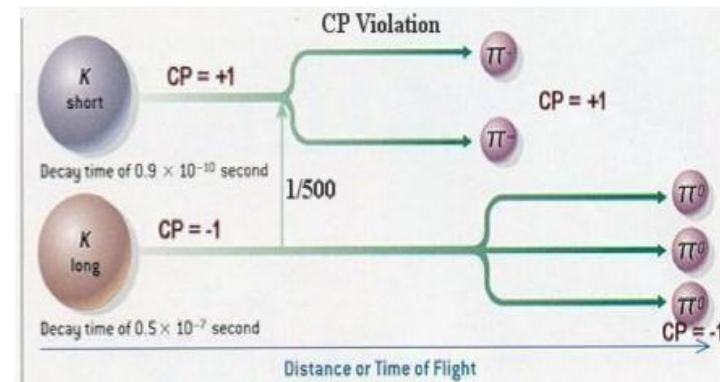
$$CP |\pi\pi\pi\rangle = - |\pi\pi\pi\rangle \quad \text{CP odd}$$

- ▶ assuming CP invariance in the decays thus yields:

CP even: $K_1 \rightarrow \pi\pi$, $K_1 \not\rightarrow \pi\pi\pi$

CP odd: $K_2 \rightarrow \pi\pi\pi$, $K_2 \not\rightarrow \pi\pi$

- ▶ we identified $|K_S\rangle = |K_1\rangle$ & $|K_L\rangle = |K_2\rangle$



- ▶ conclusion: observation of $K_L \rightarrow \pi^+ \pi^-$ events implies that K_L is not a pure CP-eigenstate \Rightarrow **(indirect) CP violation!**

CP is violated: What next ?

Hopes for CP conservation were dashed in 1964 by a Princeton group led by Val Fitch and Jim Cronin, who detected a tiny CP violating effect in neutral K decays.

This is a wonderful story, but one that we won't go into here.

Reasons for further study:

- CP violation is “surprising”
- CP violation represents a matter-antimatter asymmetry (we'll see how later on) and is an essential element in understanding the baryon-antibaryon asymmetry in Universe.
- CP effects involving b quarks are expected to be large.

Time reversal

Two basic symmetries C and P are violated maximally in weak interactions !!

A third appealing symmetry is time reversal T

Time Reversal

In non-relativistic QM, the time-reversal operator is such that: $i \rightarrow -i$ & $t \rightarrow -t$

$$T|f\rangle = |f^*\rangle$$

thus

$$\Psi(x, t) = \Psi_0 e^{-i(kx - \omega t)} \rightarrow \Psi_0^* e^{i(kx + \omega t)}$$

left-mover

right-mover

As one would expect, the T operator reverses momenta (but not positions).

CPT

- Certainly there is a very strong reason for requiring the combination of all three to be a symmetry of nature !!
 - ✓ It has been proven that any Lorentz invariant local field theory must have the combined CPT symmetry.

The easiest way of conserving the CPT invariance would clearly have been the invariance of physics to all three symmetries separately.

1. One of the consequences of the CPT symmetry is that particle states i.e. mass eigenstates will have an equivalent antiparticle mass eigenstate with the same mass eigenvalue.

2. Also, they will also have opposite charges and magnetic moments

The CPT Theorem

All that is left is an operator called CPT, where “T” stands for time-reversal.

Although the experimental tests of CPT are somewhat limited, the CPT theorem is part of the “theoretical bedrock” of field theory. If we assume that CPT is a good symmetry, then

$$\cancel{CP} \Rightarrow \cancel{T}$$

The validity of the CPT theorem is tested by looking for a difference between particles and antiparticles in terms of mass or lifetime. At the moment the best measurement of the validity of the CPT theorem comes from the measurement in the neutral kaon system of $|m_{K^0} - m_{\bar{K}^0}|/m_{K^0}$ which is smaller than one part in 10^{18}

QED

The spinor representation of the Dirac equations embodies the electrodynamic and kinematic properties of the electron like the charge/current density and the components of the spin density.

All these values can be extracted using **4x4 gamma** matrices in various combinations.

The results are the so called bilinear covariant fields, The Lorentz scalar, pseudoscalar, vector and axial vector of the theory.

In general, there are only 5 possible combinations of two spinors and the gamma matrices that are Lorentz invariant, called “bilinear covariants”:

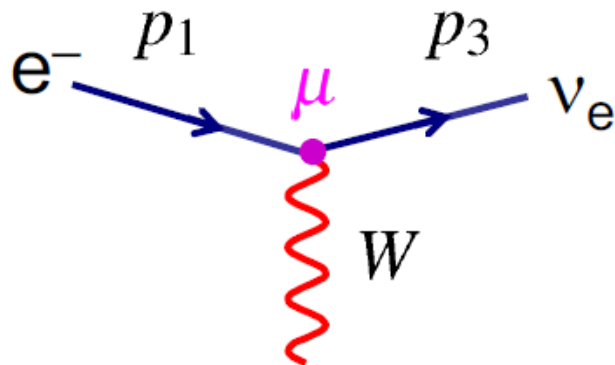
Bilinear expression		transforms like a:
$\bar{\psi}\psi$	1×	scalar
$\bar{\psi}\gamma^\mu\psi$	4×	vector
$\bar{\psi}\sigma^{\mu\nu}\psi$	6×	antisymmetric tensor
$\bar{\psi}\gamma^\mu\gamma^5\psi$	4×	axial vector
$\bar{\psi}\gamma^5\psi$	1×	pseudoscalar

➡ 16 different 4x4 matrices

The bilinear covariants of the 4d Dirac equation are listed

	Bilinear	P	C	T	CP	CPT
scalar	$\bar{\psi}_1 \psi_2$	$\bar{\psi}_1 \psi_2$	$\bar{\psi}_2 \psi_1$	$\bar{\psi}_1 \psi_2$	$\bar{\psi}_2 \psi_1$	$\bar{\psi}_2 \psi_1$
pseudo scalar	$\bar{\psi}_1 \gamma_5 \psi_2$	$-\bar{\psi}_1 \gamma_5 \psi_2$	$\bar{\psi}_2 \gamma_5 \psi_1$	$-\bar{\psi}_1 \gamma_5 \psi_2$	$-\bar{\psi}_2 \gamma_5 \psi_1$	$\bar{\psi}_2 \gamma_5 \psi_1$
vector	$\bar{\psi}_1 \gamma_\mu \psi_2$	$\bar{\psi}_1 \gamma^\mu \psi_2$	$-\bar{\psi}_2 \gamma_\mu \psi_1$	$\bar{\psi}_1 \gamma^\mu \psi_2$	$-\bar{\psi}_2 \gamma^\mu \psi_1$	$-\bar{\psi}_2 \gamma_\mu \psi_1$
axial vector	$\bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2$	$-\bar{\psi}_1 \gamma^\mu \gamma_5 \psi_2$	$\bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1$	$\bar{\psi}_1 \gamma^\mu \gamma_5 \psi_2$	$-\bar{\psi}_2 \gamma^\mu \gamma_5 \psi_1$	$-\bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1$
tensor	$\bar{\psi}_1 \sigma_{\mu\nu} \psi_2$	$\bar{\psi}_1 \sigma^{\mu\nu} \psi_2$	$-\bar{\psi}_2 \sigma_{\mu\nu} \psi_1$	$-\bar{\psi}_1 \sigma^{\mu\nu} \psi_2$	$-\bar{\psi}_2 \sigma^{\mu\nu} \psi_1$	$\bar{\psi}_2 \sigma_{\mu\nu} \psi_1$

- ★ The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants
- ★ For an interaction corresponding to the exchange of a spin-1 particle the most general form is a linear combination of **VECTOR** and **AXIAL-VECTOR**
- ★ The form for WEAK interaction is determined from experiment to be **VECTOR – AXIAL-VECTOR** (V – A)



$$j^\mu \propto \bar{u}_{\nu_e} (\gamma^\mu - \gamma^\mu \gamma^5) u_e$$

V – A

- ★ Can this account for parity violation?

Experimental evidence for V-A

- ★ The V-A nature of the charged current weak interaction vertex fits with experiment

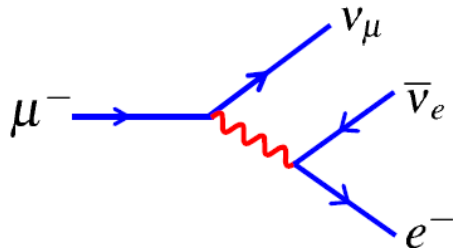
EXAMPLE charged pion decay

- Experimentally measure: $\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = (1.230 \pm 0.004) \times 10^{-4}$
- Theoretical predictions (depend on Lorentz Structure of the interaction)

V-A $(\bar{\psi}\gamma^\mu(1-\gamma^5)\phi)$ or **V+A** $(\bar{\psi}\gamma^\mu(1+\gamma^5)\phi)$ $\rightarrow \frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \approx 1.3 \times 10^{-4}$

Scalar $(\bar{\psi}\phi)$ or **Pseudo-Scalar** $(\bar{\psi}\gamma^5\phi)$ $\rightarrow \frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = 5.5$

EXAMPLE muon decay

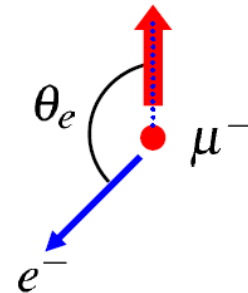


e.g. TWIST expt: 6×10^9 μ decays
Phys. Rev. Lett. 95 (2005) 101805

Measure **electron** energy and angular distributions relative to muon spin direction. Results expressed in terms of general **S+P+V+A+T** form in “Michel Parameters”

$$\rho = 0.75080 \pm 0.00105$$

V-A Prediction: $\rho = 0.75$



Can V-A interaction violates P ?

- The space-like components remain unchanged and the time-like components change sign (the opposite to the parity properties of a vector-current)

$$j_A^0 \xrightarrow{\hat{P}} -j_A^0; \quad j_A^k \xrightarrow{\hat{P}} +j_A^k; \quad j_V^0 \xrightarrow{\hat{P}} +j_V^0; \quad j_V^k \xrightarrow{\hat{P}} -j_V^k$$

- Now consider the matrix elements

$$M \propto g_{\mu\nu} j_1^\mu j_2^\nu = j_1^0 j_2^0 - \sum_{k=1,3} j_1^k j_2^k$$

- For the combination of a two axial-vector currents

$$j_{A1} \cdot j_{A2} \xrightarrow{\hat{P}} (-j_1^0)(-j_2^0) - \sum_{k=1,3} (j_1^k)(j_2^k) = j_{A1} \cdot j_{A2}$$

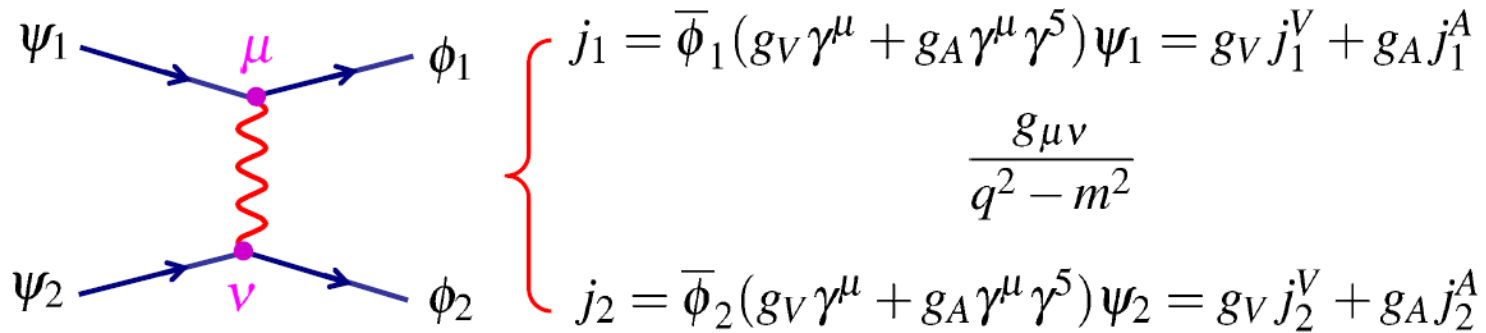
- Consequently parity is conserved in a pure vector and pure axial-vector interactions

- However the combination of a vector current and an axial vector current

$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} (j_1^0)(-j_2^0) - \sum_{k=1,3} (-j_1^k)(j_2^k) = -j_{V1} \cdot j_{A2}$$

changes sign under parity – can give parity violation !

- ★ Now consider a general linear combination of VECTOR and AXIAL-VECTOR
(note this is relevant for the Z-boson vertex)



$$j_1 = \bar{\phi}_1 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_1 = g_V j_1^V + g_A j_1^A$$

$$j_2 = \bar{\phi}_2 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_2 = g_V j_2^V + g_A j_2^A$$

$$\frac{g_{\mu\nu}}{q^2 - m^2}$$

$$M_{fi} \propto j_1 \cdot j_2 = g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A + g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- Consider the parity transformation of this scalar product

$$j_1 \cdot j_2 \xrightarrow{\hat{P}} g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A - g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- If either g_A or g_V is zero, Parity is conserved, i.e. parity conserved in a pure VECTOR or pure AXIAL-VECTOR interaction

- Relative strength of parity violating part $\propto \frac{g_V g_A}{g_V^2 + g_A^2}$

Maximal Parity Violation for V-A (or V+A)



- ★ Consequently only left-handed chiral particle states and right-handed chiral anti-particle states participate in the weak interaction

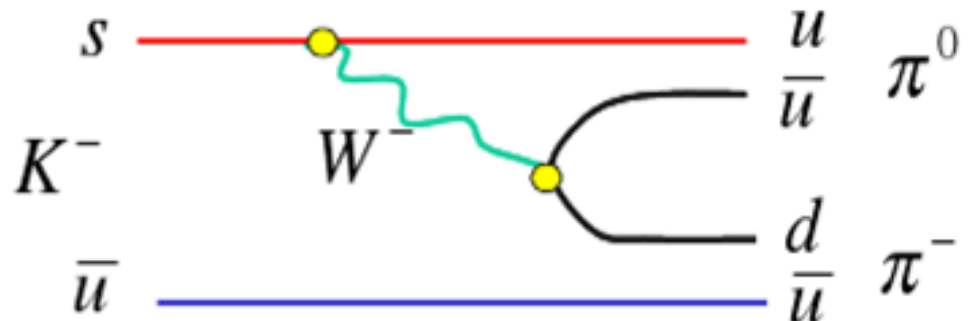
Quark mixing

Experimentally we know that the eigenstates of the weak Hamiltonian and the mass eigenstates are different. For simplicity we start with a two-quark-doublet version of nature, i.e.,

$$\begin{array}{l} q = +\frac{2}{3} \\ q = -\frac{1}{3} \end{array} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix}$$

If the quarks acted like leptons, then only vertical transitions would be allowed and the s quark would be stable.

However, the kaon decays in 12 ns. It appears that there are generation-crossing transitions.



Quark mixing : CP violation

Rather than saying that the strange quark is decaying directly to an up quark, we write the following

Cabibbo mixing

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

And say that the s-quark in the kaon has a d' component that can decay into a u-quark.

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$

Weak eigenstate

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

Mass eigenstate

Q: What does this have to do with CP violation?

Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{kinetic} = i\bar{\psi}(D^\mu\gamma_\mu)\psi \quad D^\mu = \partial^\mu + ig_s G_a^\mu L_a + ig W_b^\mu \sigma_b + ig' B^\mu Y,$$

with L_a the Gell-Mann matrices and σ_b the Pauli matrices. G_a^μ , W_b^μ and B^μ are the eight gluon fields, the three weak interaction bosons and the single hypercharge boson, respectively.

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \quad \phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with ϕ an isospin doublet

$$\begin{aligned} -\mathcal{L}_{Yukawa} &= Y_{ij}\bar{\psi}_{Li}\phi\psi_{Rj} + h.c. \\ &= Y_{ij}^d\bar{Q}_{Li}^I\phi d_{Rj}^I + Y_{ij}^u\bar{Q}_{Li}^I\tilde{\phi}u_{Rj}^I + Y_{ij}^l\bar{L}_{Li}^I\phi l_{Rj}^I + h.c. \end{aligned}$$

with

$$\tilde{\phi} = i\sigma_2\phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}.$$

The matrices Y_{ij}^d , Y_{ij}^u and Y_{ij}^l are arbitrary complex matrices that operate in flavour space
couplings between different families, or quark mixing

Mass term

After spontaneous symmetry breaking,

$$\phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\text{sym. breaking}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix},$$

the following mass terms for the fermion fields arise:

$$\begin{aligned} -\mathcal{L}_{Yukawa}^{quarks} &= Y_{ij}^d \overline{Q_{Li}^I} \phi d_{Rj}^I + Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + h.c. \\ &= Y_{ij}^d \overline{d_{Li}^I} \frac{v}{\sqrt{2}} d_{Rj}^I + Y_{ij}^u \overline{u_{Li}^I} \frac{v}{\sqrt{2}} u_{Rj}^I + h.c. + \text{interaction terms} \\ &= M_{ij}^d \overline{d_{Li}^I} d_{Rj}^I + M_{ij}^u \overline{u_{Li}^I} u_{Rj}^I + h.c. + \text{interaction terms} \end{aligned}$$

To obtain proper mass terms, the matrices M^d and M^u should be diagonalized. We do this with unitary matrices V^d as follows:

$$\begin{aligned} M_{diag}^d &= V_L^d M^d V_R^{d\dagger} \\ M_{diag}^u &= V_L^u M^u V_R^{u\dagger} \end{aligned}$$

Mass term

Using the requirement that the matrices V are unitary ($V_L^{d\dagger} V_L^d = \mathbb{1}$) the Lagrangian can now be expressed as follows:

$$-\mathcal{L}_{Yukawa}^{quarks} = \overline{d_{Li}} (M_{ij}^d)_{diag} d_{Rj} + \overline{u_{Li}} (M_{ij}^u)_{diag} u_{Rj} + h.c. + \dots$$

Mass eigenstates



$$\begin{aligned} d_{Li} &= (V_L^d)_{ij} d_{Lj}^I & d_{Ri} &= (V_R^d)_{ij} d_{Rj}^I \\ u_{Li} &= (V_L^u)_{ij} u_{Lj}^I & u_{Ri} &= (V_R^u)_{ij} u_{Rj}^I \end{aligned} \quad \xrightarrow{\text{interaction eigenstates}}$$

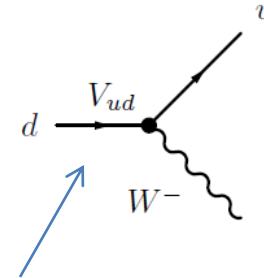
If we now express the Lagrangian in terms of the quark mass eigenstates d , u instead of the weak interaction eigenstates d^I , u^I , the price to pay is that the quark mixing between families (i.e. the off-diagonal elements) appears in the charged current interaction:

$$\begin{aligned} \mathcal{L}_{kinetic,cc}(Q_L) &= \frac{g}{\sqrt{2}} \overline{u_{iL}^I} \gamma_\mu W^{-\mu} d_{iL}^I + \frac{g}{\sqrt{2}} \overline{d_{iL}^I} \gamma_\mu W^{+\mu} u_{iL}^I + \dots \\ &= \frac{g}{\sqrt{2}} \overline{u_{iL}} (V_L^u V_L^{d\dagger})_{ij} \gamma_\mu W^{-\mu} d_{iL} + \frac{g}{\sqrt{2}} \overline{d_{iL}} (V_L^d V_L^{u\dagger})_{ij} \gamma_\mu W^{+\mu} u_{iL} + \dots \end{aligned}$$

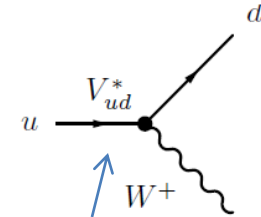
$$V_{CKM} = (V_L^d V_L^{u\dagger})_{ij} \quad \Rightarrow \quad \text{Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix}$$

CKM & CP violation

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Transition for down to up



Transition from up to down

CP violation shows up in the complex Yukawa couplings

$$\begin{aligned} -\mathcal{L}_{Yukawa} &= Y_{ij} \overline{\psi_{Li}} \phi \psi_{Rj} + h.c. \\ &= Y_{ij} \overline{\psi_{Li}} \phi \psi_{Rj} + Y_{ij}^* \overline{\psi_{Rj}} \phi^\dagger \psi_{Li} \end{aligned}$$

$CP(\overline{\psi_{Li}} \phi \psi_{Rj}) = \overline{\psi_{Rj}} \phi^\dagger \psi_{Li}$

So, \mathcal{L}_{Yukawa} remains unchanged under the CP operation if $Y_{ij} = Y_{ij}^*$.

Charged current coupling

$$\mathcal{L}_{kinetic,cc}(Q_L) = \frac{g}{\sqrt{2}} \overline{u_{iL}} V_{ij} \gamma_\mu W^{-\mu} d_{iL} + \frac{g}{\sqrt{2}} \overline{d_{iL}} V_{ij}^* \gamma_\mu W^{+\mu} u_{iL}$$

Under CP

$$\mathcal{L}_{kinetic,cc}^{CP}(Q_L) = \frac{g}{\sqrt{2}} \overline{d_{iL}} V_{ij} \gamma_\mu W^{+\mu} u_{iL} + \frac{g}{\sqrt{2}} \overline{u_{iL}} V_{ij}^* \gamma_\mu W^{-\mu} d_{iL}$$

The complex nature of CKM is the origin of CP violation in the SM !!

← Lagrangian is unchanged if $V_{ij} = V_{ij}^*$.

Properties of CKM

Number of free parameters for the unitary CKM matrix

- 1) A general $n \times n$ complex matrix has n^2 complex elements, and thus $2n^2$ real parameters.
- 2) Unitarity ($V^\dagger V = \mathbb{1}$) implies n^2 constraints:
 - n unitary conditions (unity of the diagonal elements);
 - $n^2 - n$ orthogonality relations (vanishing off-diagonal elements).
- 3) The phases of the quarks can be rotated freely: $u_{Li} \rightarrow e^{i\phi_i^u} u_{Li}$ and $d_{Lj} \rightarrow e^{i\phi_j^d} d_{Lj}$. Since the overall phase is irrelevant, $2n - 1$ relative quark phases can be removed.

Free parameters



$$2n^2 - n^2 - (2n - 1) = (n - 1)^2$$

➡ A general $n \times n$ orthogonal matrix can be constructed from $\frac{1}{2}n(n - 1)$ angles describing the rotations among the n dimensions.

➡ The remaining free parameters are the phases: $(n - 1)^2 - \frac{1}{2}n(n - 1) = \frac{1}{2}(n - 1)(n - 2)$

CP violation with three generation

Quark Mixing

Case	N_{gen}	Parameter(s)
Cabibbo	2×2	θ_C
KM	3×3	$\theta_1, \theta_2, \theta_3, e^{i\delta}$

The essential contribution of Kobayashi and Maskawa was the observation that only a 3x3 scheme would provide the phase needed for T violation (and hence CP violation).

Parameterizations

In the literature there are many different parameterizations of the CKM matrix

Particle Data Group: by Chau and Keung,

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$

The phase can be made to appear in many elements, and is chosen here to appear in the matrix describing the relation between the 1st and 3rd family.

Lec-2

Size of the elements !

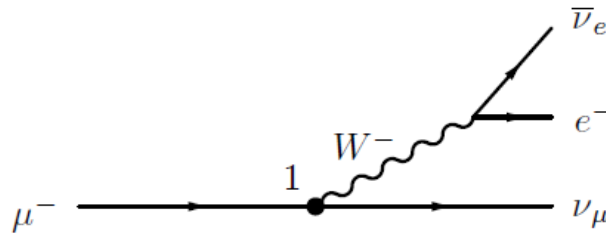
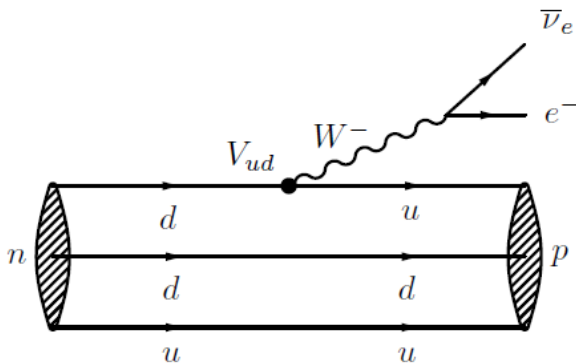
$|V_{ud}|$:

The magnitude of V_{ud} is measured in weak $u \leftrightarrow d$ transitions

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu (\gamma)) = \frac{G_F^2}{8\pi} f_\pi^2 m_\mu^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 |V_{ud}|^2 \left(1 + \frac{\alpha}{\pi} C_\pi\right)$$

Pion decay constant

radiative corrections



Most precise value come from comparing the nuclear β - decay rate to μ decay rates !!

$$|V_{ud}| = 0.97418 \pm 0.00027$$

Size ...CKM

$|V_{us}|$:

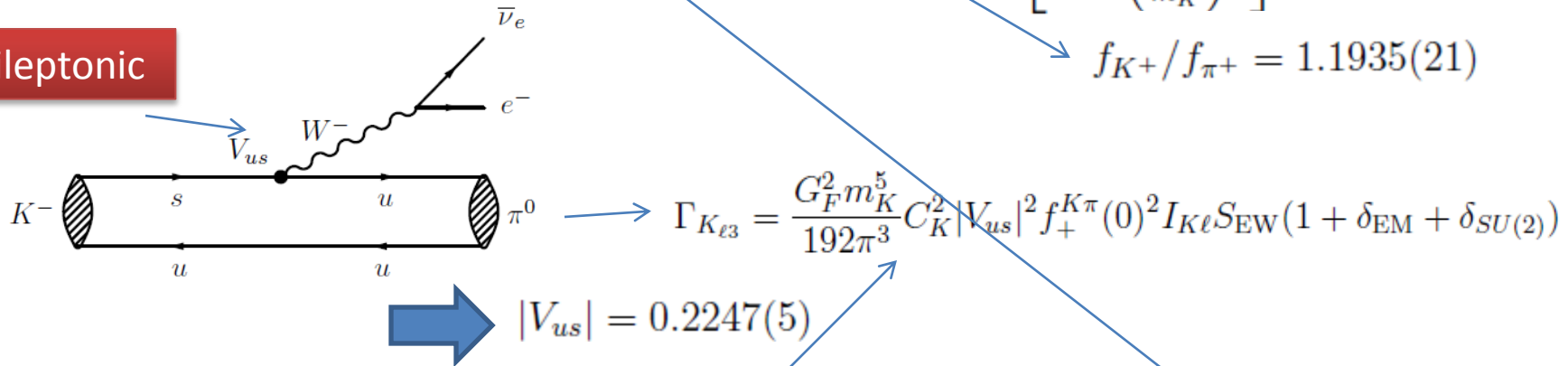
The measurement of $|V_{us}|$ could be done from purely leptonic or semileptonic decays of Kaon

Purely leptonic decay

$$|V_{us}|^2 = |V_{ud}|^2 \frac{\Gamma(K \rightarrow \mu\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \left(\frac{f_\pi}{f_K}\right)^2 \frac{m_\pi}{m_K} \left[\frac{1 - \left(\frac{m_\mu}{m_\pi}\right)^2}{1 - \left(\frac{m_\mu}{m_K}\right)^2} \right] \frac{1 + \frac{\alpha}{\pi} C_\pi}{1 + \frac{\alpha}{\pi} C_K}$$

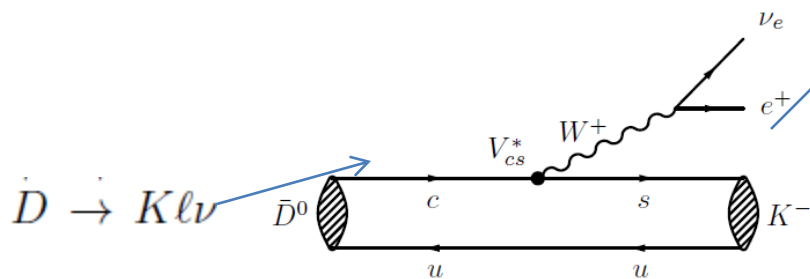
$$f_{K^+}/f_{\pi^+} = 1.1935(21)$$

Semileptonic



$|V_{cs}|$:

The $c \rightarrow s$ transition is the “Cabibbo-favored” decay channel for charm



Alternative channel

$$D_s \rightarrow \ell \nu$$

Analogous to $|V_{us}|$ measurement

$$|V_{cs}| = 0.992(15)$$

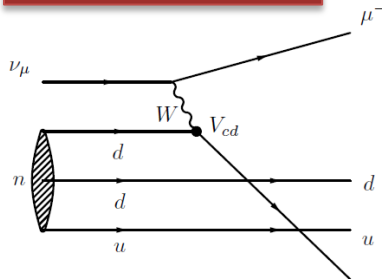
Size ...CKM

$|V_{cd}|$:

Completing the upper left 2×2 submatrix is the “Cabibbo-suppressed” $c \rightarrow d$ transition

Main channel

Neutrino and anti-neutrino induced charm production of the valence d-quark in neutron !!



$$|V_{cd}| = 0.216(5)$$

Advancement of lattice calculation allows the extraction from semileptonic decays

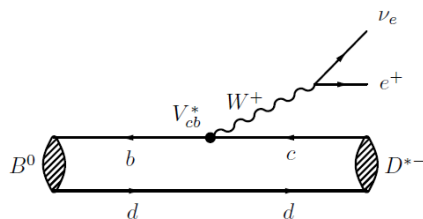
$D \rightarrow \pi \ell \nu$ decay is also useful, analogous to $D \rightarrow K \ell \nu$ for $|V_{cs}|$

$|V_{cb}|$:

Inclusive and exclusive semileptonic B decays to charm

$$\frac{d\Gamma(\bar{B} \rightarrow D^{(*)} \ell \nu)}{dw} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^{(*)}}^3 \sqrt{w^2 - 1} \eta_{EW}^2 \begin{cases} (m_B + m_{D^{(*)}})^2 (w^2 - 1) G(w)^2 & D, \\ (m_B - m_{D^{(*)}})^2 (w + 1)^2 \left[1 + \frac{4w}{w+1} \frac{1-2wr+r^2}{(1-r)^2} \right] F(w)^2 & D^* \end{cases}$$

provides measurements of $|V_{cb}|G(w)$ and $|V_{cb}|F(w)$



In the B rest frame $w = E_{D^{(*)}}/m_{D^{(*)}}$

$$r = m_{D^*}/m_B$$

$$|V_{cb}|(\text{excl}) = 0.0392(5)(3)$$

$$|V_{cb}|(\text{incl}) = 0.0422(3)(7)$$

CKM ...size

$|V_{ub}|$: The measurement of $|V_{ub}|$ is analogous with the measurement of $|V_{cb}|$

Difficult due to smaller branching fraction and larger relative background

Again both Inclusive and exclusive approaches are taken !!

→ So far, the most precise exclusive measurement of $|V_{ub}|$ is obtained with $B \rightarrow \pi \ell \nu$

Channel	$ V_{ub} $
$B \rightarrow \pi \ell \nu$	0.00372(10)(12)
$\Lambda_b \rightarrow p \mu \nu$	0.00325(16)(16)
$B \rightarrow \tau \nu$	0.00422(40)(9)

Theoretically challenging because of the large $b \rightarrow c$ inclusive decays !!

2014 average

Present average

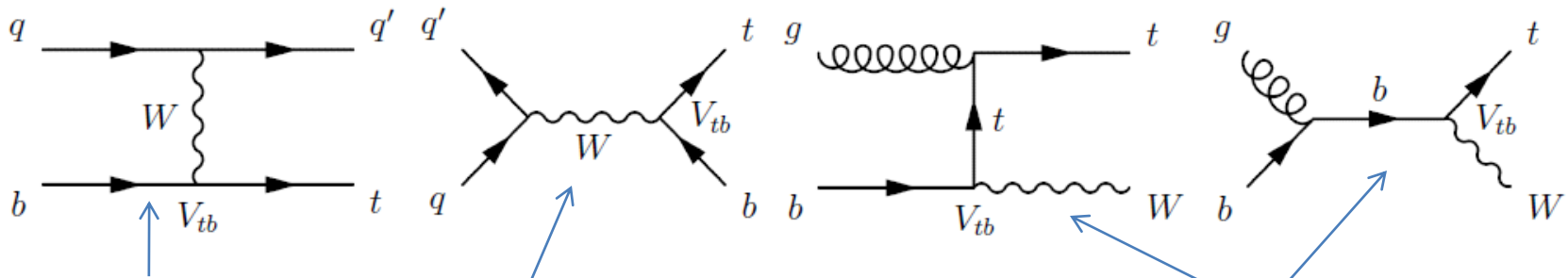
	2014 average	Present average
ADFR	$4.05 \pm 0.13^{+0.18}_{-0.11}$	$4.42 \pm 0.19 \pm 0.19$
BLNP	$4.45 \pm 0.15^{+0.20}_{-0.21}$	$4.40 \pm 0.18 \pm 0.21$
DGE	$4.52 \pm 0.16^{+0.15}_{-0.16}$	$4.53 \pm 0.18 \pm 0.13$
GGOU	$4.51 \pm 0.16^{+0.12}_{-0.15}$	$4.50 \pm 0.18 \pm 0.11$

leptons above the endpoint for $b \rightarrow c$ transitions

$|V_{tb}|$

Single top production

The production cross section is proportional to $|V_{tb}|^2$



t-channel

s-channel

Wt associated production

The theoretical cross section is computed at next-to-next-to-leading-order (NNLO) in QCD

$$|V_{tb}| = 1.02^{+0.06}_{-0.05}$$

100% correlation in the theoretical cross sections!!

$$|V_{tb}| = 1.007(36)$$

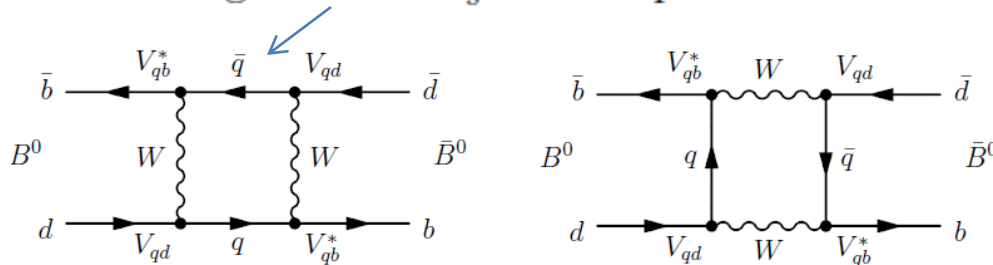
CKM from loop induced processes

$|V_{td}|$ and $|V_{ts}|$: The remaining third-row elements V_{td} and V_{ts} are very small

Experimentally difficult to precisely measure the $t \rightarrow d$ or $t \rightarrow s$ cross section in single top production !

Instead, these elements are currently best measured in virtual processes involving loop diagrams

The mixing of B^0 or B_s^0 mesons provides for measuring V_{td} or V_{ts}



$$\Delta m = \frac{G_F^2 m_W^2 m_B}{6\pi^2} \eta_B f_B^2 \hat{B}_B S_0(x_t) |V_{tq} V_{tb}|^2$$

Feynman graphs of box diagrams for $B^0 - \bar{B}^0$ mixing

$$|V_{td}| = 0.0084(7),$$

$$|V_{ts}| = 0.0401(31)$$

Hierarchy in CKM elements

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},$$

➡ The strength of the charged current couplings seem to exhibit a hierarchy.

This pattern motivated Wolfenstein to parameterize the CKM elements in powers of parameter $\lambda \approx \sin(\theta_{12})$

$$|V_{CKM}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\sin \theta_{12} = \lambda$$

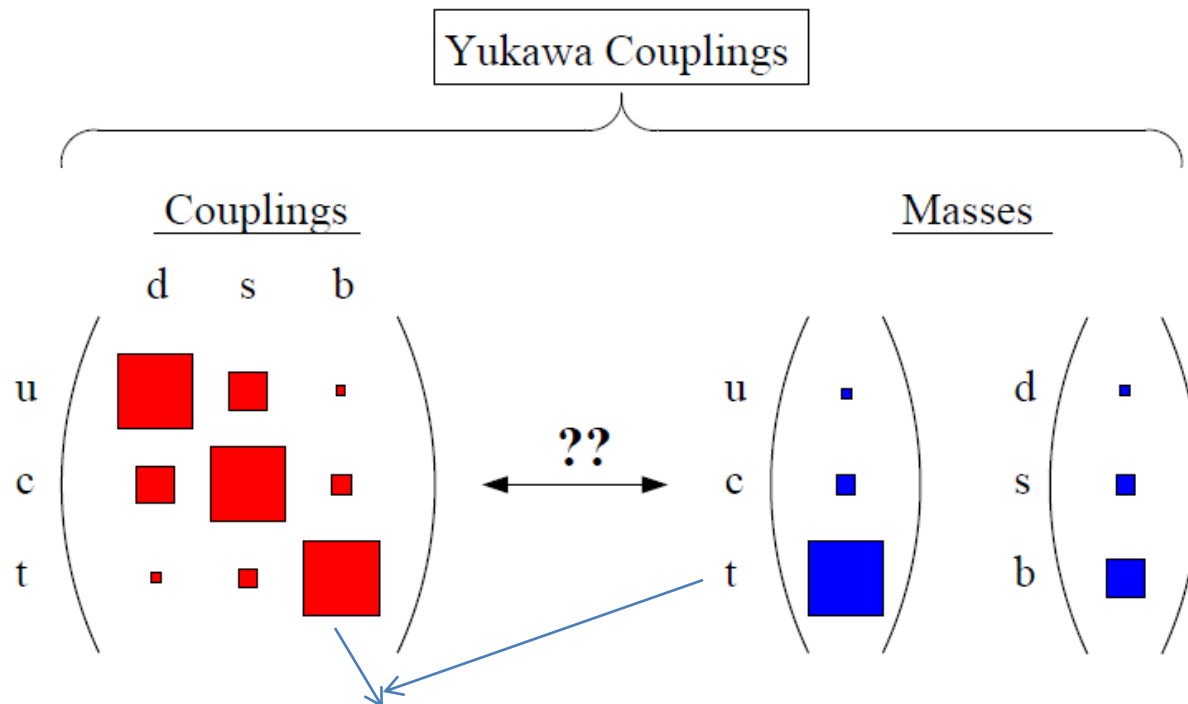
$$\sin \theta_{23} = A\lambda^2$$

$$\sin \theta_{13} e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta)$$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Quark masses \leftrightarrow CC couplings

- ✓ We have seen that the origin of quark mixing matrix lies in the Yukawa coupling !!
- ✓ These Yukawa couplings are responsible for the generation of quark masses !!



Both the charged current quark couplings and the quark masses show an intriguing hierarchy !



Does this suggest an underlying connection between them ??

Unitary Triangle

Let us now return to the six orthogonality relations that give rise to the six unitarity triangles. Only two out of the six equations have terms with equal powers in λ .

$$\begin{array}{l} V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \\ \mathcal{O}(\lambda^3) \quad \mathcal{O}(\lambda^3) \quad \mathcal{O}(\lambda^3) \end{array}$$

$$\begin{array}{l} V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* = 0 \\ \mathcal{O}(\lambda^3) \quad \mathcal{O}(\lambda^3) \quad \mathcal{O}(\lambda^3) \end{array}$$

Relevant for B decays

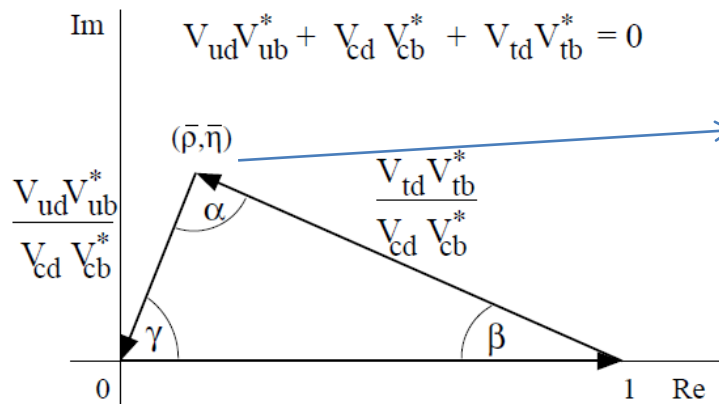
Known as Unitarity Triangle

convention independent

$$\alpha \equiv \arg \left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right]$$

$$\beta \equiv \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right]$$

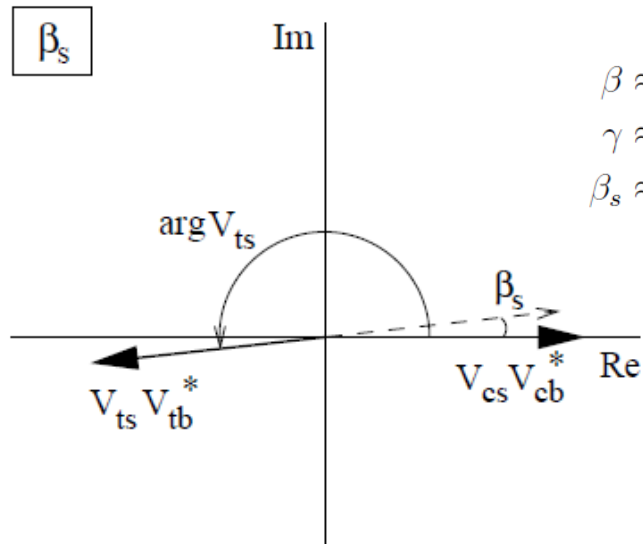
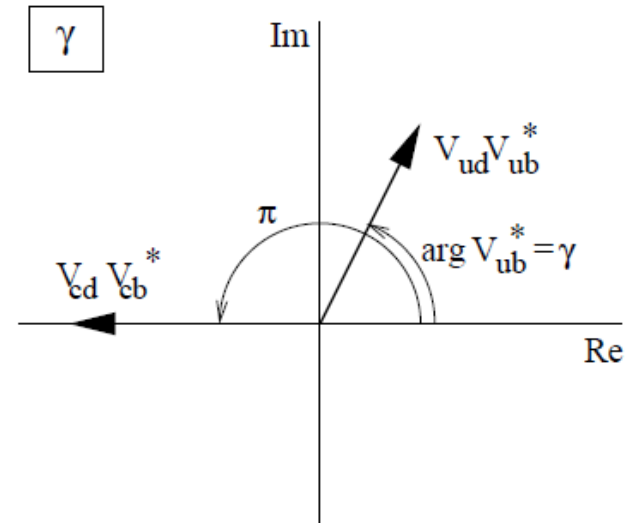
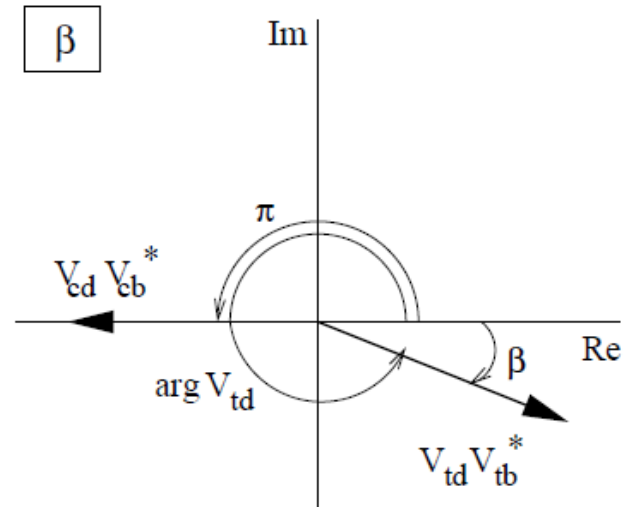
$$\gamma \equiv \arg \left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]$$



$$\bar{\rho} + i\bar{\eta} \equiv \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$$\bar{\rho} = \rho \left(1 - \frac{1}{2} \lambda^2 \right) + \mathcal{O}(\lambda^4) \quad \bar{\eta} = \eta \left(1 - \frac{1}{2} \lambda^2 \right) + \mathcal{O}(\lambda^4)$$

Phase conventions



$$\begin{aligned}\beta &\approx \pi + \arg(V_{cd}V_{cb}^*) - \arg(V_{td}V_{tb}^*) = \pi + \pi - \arg(V_{td}) = -\arg(V_{td}) \\ \gamma &\approx \pi + \arg(V_{ud}V_{ub}^*) - \arg(V_{cd}V_{cb}^*) = \pi - \arg(V_{ub}) - \pi = -\arg(V_{ub}) \\ \beta_s &\approx \pi + \arg(V_{ts}V_{tb}^*) - \arg(V_{cs}V_{cb}^*) = \pi + \arg(V_{ts}) - 0 = \arg(V_{ts}) + \pi\end{aligned}$$

$$V_{CKM, \text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

Jarlskog invariant

As mentioned earlier, CP violation requires $V_{ij} \neq V_{ij}^*$

- ✓ Satisfied if the triangle has a finite area in the complex plan !
- ✓ Area of all the six unitarity triangles are same !

➤ The corresponding quantity is known as Jarlskog invariant denoted by J

Derived in a simple way from the CKM matrix:

- ✓ Remove one column and one row !
- ✓ Take the product of the diagonal element with the complex conjugate of the non-diagonal elements !
- ✓ The imaginary part of the product is then equal to J !

$J = \Im(V_{11}V_{22}V_{12}^*V_{21}^*) = \Im(V_{22}V_{33}V_{23}^*V_{32}^*) = \dots$ There will be nine possible expressions for J which all give the same results !

In the Wolfenstein parameterization the quantity J becomes

$$J = A^2 \lambda^6 \eta = 2 \times \text{area}$$

In the earlier parameterization

$$J = c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23} \sin \delta_{13}$$

Neutral meson oscillation

- The phenomenon of neutral meson mixing plays an important role for the extraction of CKM phase !!

$$\psi(t) = a(t)|P^0\rangle + b(t)|\bar{P}^0\rangle$$

Eigenstates of electromagnetic and strong interactions !

We can write $\psi(t)$ in the subspace of P^0 and \bar{P}^0 as follows

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \quad \Rightarrow \quad i\frac{\partial\psi}{\partial t} = H\psi \quad \xrightarrow{H = M - \frac{i}{2}\Gamma}$$

M and Γ are Hermitian matrices

With the weak interactions responsible for decay !

$$i\frac{\partial\psi}{\partial t} = H\psi = \left(M - \frac{i}{2}\Gamma\right)\psi = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & 0 \\ 0 & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \psi$$

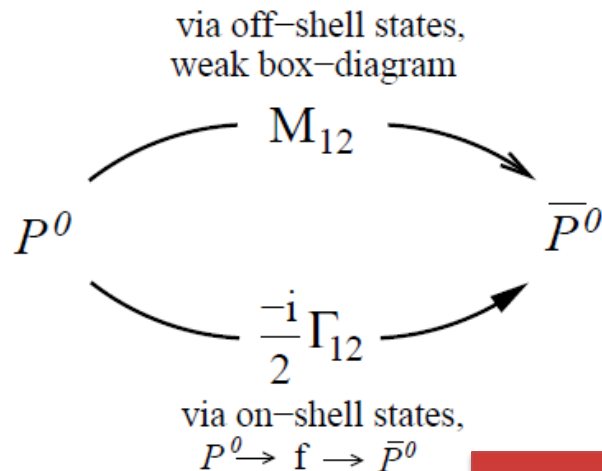
➡ If we now allow for the transitions $P^0 \rightarrow \bar{P}^0$, the off-diagonal elements are introduced:

$$i\frac{\partial\psi}{\partial t} = H\psi = \left(M - \frac{i}{2}\Gamma\right)\psi = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \psi$$

Oscillation

In case CPT is valid

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \psi$$



Contributions to neutral meson oscillation !

Under this assumptions we can now find the eigen values and eigen vectors of the Hamiltonian !!

✓ This will describe the masses and the decay width

Mass eigenstates !

Heavy and light mass eigenstate:

$$|P_L\rangle = p|P^0\rangle + q|\bar{P}^0\rangle \quad \text{with } m_L, \Gamma_L$$

$$|P_H\rangle = p|P^0\rangle - q|\bar{P}^0\rangle \quad \text{with } m_H, \Gamma_H$$

$$|p|^2 + |q|^2 = 1 \quad \text{complex coefficients}$$



$$|P^0\rangle = \frac{1}{2p}(|P_L\rangle + |P_H\rangle)$$

$$|\bar{P}^0\rangle = \frac{1}{2q}(|P_L\rangle - |P_H\rangle)$$

**Flavor/weak
eigenstates**

Parameters of the mass states

$$m_{H,L} = m \pm \text{Re} \sqrt{H_{12}H_{21}}$$

$$\Gamma_{H,L} = \Gamma \mp 2\text{Im} \sqrt{H_{12}H_{21}}$$

$$\Delta m = m_H - m_L = 2\text{Re} \sqrt{H_{12}H_{21}}$$

$$\Delta \Gamma = \Gamma_H - \Gamma_L = -4\text{Im} \sqrt{H_{12}H_{21}}$$

$$x \equiv \frac{\Delta m}{\Gamma} \quad \text{und} \quad y \equiv \frac{\Delta \Gamma}{2\Gamma}$$

Time evolution !

From the schroedinger equation we will get :

$$\begin{aligned} |P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t} |P_H(0)\rangle \\ |P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t} |P_L(0)\rangle \end{aligned}$$

Hence, $|P^0(t)\rangle = g_+(t)|P^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{P}^0\rangle$

where

$$\begin{aligned} g_+(t) &= \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) = \frac{1}{2} e^{-iMt} \left(e^{-i\frac{1}{2}\Delta m t - \frac{1}{2}\Gamma_H t} + e^{+i\frac{1}{2}\Delta m t - \frac{1}{2}\Gamma_L t} \right) \\ g_-(t) &= \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) = \frac{1}{2} e^{-iMt} \left(e^{-i\frac{1}{2}\Delta m t - \frac{1}{2}\Gamma_H t} - e^{+i\frac{1}{2}\Delta m t - \frac{1}{2}\Gamma_L t} \right) \end{aligned}$$

Likewise, $|\bar{P}^0(t)\rangle = g_-(t) \left(\frac{p}{q}\right) |P^0\rangle + g_+(t)|\bar{P}^0\rangle$

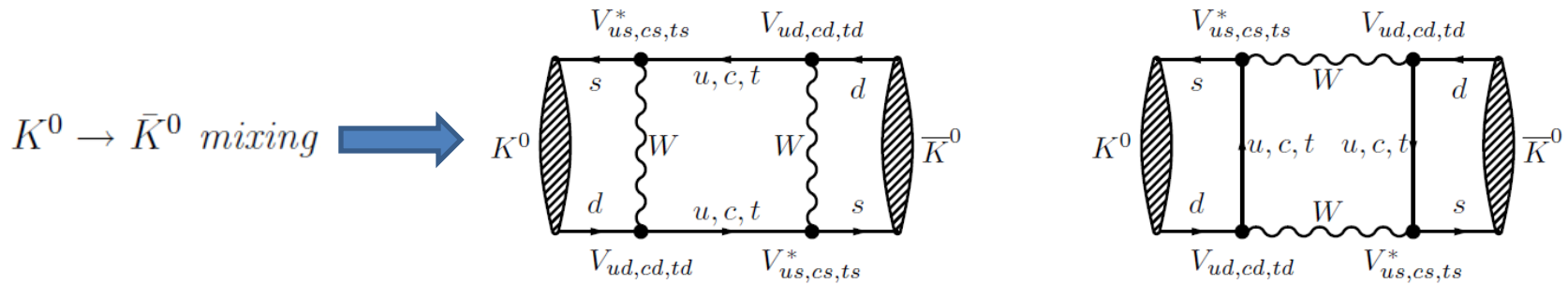
If we start from a pure sample of $|P^0\rangle$ particles (e.g. produced by the strong interaction) then we can calculate the probability of measuring the state $|\bar{P}^0\rangle$ at time t :

$$|\langle \bar{P}^0(t) | P^0 \rangle|^2 = |g_-(t)|^2 \left(\frac{p}{q}\right)^2 \quad \text{with} \quad |g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta \Gamma t \pm \cos \Delta m t \right)$$

Decay width

Δm

The short distance contribution to the $P^0 \leftrightarrow \bar{P}^0$ transitions of neutral meson oscillations is described by Δm and can be represented by a Feynman diagram known as the box diagram, and can be calculated in perturbation theory.



$$\Delta m = m_{K_S^0} - m_{K_L^0} = \langle K_S^0 | H | K_S^0 \rangle - \langle K_L^0 | H | K_L^0 \rangle$$

$$\begin{aligned} \mathcal{M}_{uu} &= i \left(\frac{-ig_w}{2\sqrt{2}} \right)^4 (V_{us}^* V_{ud} V_{us}^* V_{ud}) \\ &\int \frac{d^4 k}{(2\pi)^4} \left(\frac{-ig^{\lambda\sigma} - k^\lambda k^\sigma / m_W^2}{k^2 - m_W^2} \right) \left(\frac{-ig^{\alpha\rho} - k^\alpha k^\rho / m_W^2}{k^2 - m_W^2} \right) \\ &\left[\bar{u}_s \gamma_\lambda (1 - \gamma^5) \frac{\not{k} + m_u}{k^2 - m_u^2} \gamma_\rho (1 - \gamma^5) u_d \right] \left[\bar{v}_s \gamma_\alpha (1 - \gamma^5) \frac{\not{k} + m_u}{k^2 - m_u^2} \gamma_\sigma (1 - \gamma^5) v_d \right] \end{aligned}$$

Δm_K

Taking the sum of all amplitudes with all possible intermediate quark lines we get an amplitude which is proportional to (assuming $k^2 \ll m_W^2$).

$$\mathcal{M} \propto \int d^4k \, k_\mu k_\nu \left(\frac{V_{us}^* V_{ud}}{k^2 - m_u^2} + \frac{V_{cs}^* V_{cd}}{k^2 - m_c^2} + \frac{V_{ts}^* V_{td}}{k^2 - m_t^2} \right)^2$$

Using unitarity condition $V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$

$$\mathcal{M} \propto \int d^4k \, k_\mu k_\nu \left(V_{cs}^* V_{cd} \left[\frac{1}{k^2 - m_c^2} - \frac{1}{k^2 - m_u^2} \right] + V_{ts}^* V_{td} \left[\frac{1}{k^2 - m_t^2} - \frac{1}{k^2 - m_u^2} \right] \right)^2$$

$$\Delta m_K = \frac{G_F^2 m_W^2}{6\pi^2} \eta_{QCD} B_K f_K^2 m_K \left[S_0(m_c^2/m_W^2) |V_{cd} V_{cs}|^2 \right]$$

Inami-Lim function

where G_F is the Fermi coupling constant, η_{QCD} is the QCD correction (≈ 0.85), B and f_K^2 is the “bag-factor” and the decay constant, respectively, which describe the effect of the transition from bound to free quarks and V_{ij} are the CKM matrix elements.

Meson decays

Decay of meson to a final state f

$$\begin{aligned} A(f) &= \langle f|T|P^0\rangle & \bar{A}(f) &= \langle f|T|\bar{P}^0\rangle \\ A(\bar{f}) &= \langle \bar{f}|T|P^0\rangle & \bar{A}(\bar{f}) &= \langle \bar{f}|T|\bar{P}^0\rangle \end{aligned}$$

we define

$$\left\{ \begin{aligned} \lambda_f &= \frac{q}{p} \frac{\bar{A}_f}{A_f}, & \bar{\lambda}_f &= \frac{1}{\lambda_f} \\ \lambda_{\bar{f}} &= \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, & \bar{\lambda}_{\bar{f}} &= \frac{1}{\lambda_{\bar{f}}} \end{aligned} \right\}$$

Hence $\Gamma_{P^0 \rightarrow f}(t) = |\langle f|T|P^0(t)\rangle|^2$

Probability that a state P^0 at time $t=0$ will decay to a final state f at time t

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

$$\Gamma_{P^0 \rightarrow \bar{f}}(t) = |\bar{A}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 \left(|g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_+(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+(t) g_-^*(t)] \right)$$

$$\Gamma_{\bar{P}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 \left(|g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\Re[\lambda_f g_+(t) g_-^*(t)] \right)$$

$$\Gamma_{\bar{P}^0 \rightarrow \bar{f}}(t) = |\bar{A}_{\bar{f}}|^2 \left(|g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_-(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+^*(t) g_-(t)] \right)$$

$$\begin{aligned} g_+^*(t) g_-(t) &= \frac{e^{-\Gamma t}}{2} \left(\sinh \frac{1}{2} \Delta \Gamma t + i \sin \Delta m t \right) \\ g_+(t) g_-^*(t) &= \frac{e^{-\Gamma t}}{2} \left(\sinh \frac{1}{2} \Delta \Gamma t - i \sin \Delta m t \right) \end{aligned}$$

Neutral Meson

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t + C_f \cos \Delta m t - S_f \sin \Delta m t \right)$$

$$\Gamma_{\bar{P}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t - C_f \cos \Delta m t + S_f \sin \Delta m t \right)$$

$$D_f = \frac{2\Re\lambda_f}{1 + |\lambda_f|^2}$$

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

$$S_f = \frac{2\Im\lambda_f}{1 + |\lambda_f|^2}$$

For a given final state we have to find out only λ_f to fully describe the decay (oscillating) meson !!

Classification of the CP violating effects !

CP violation in decay !

This type of CP violation occurs when

$$\Gamma(P^0 \rightarrow f) \neq \Gamma(\bar{P}^0 \rightarrow \bar{f})$$

This is obvious when $\left| \frac{\bar{A}_{\bar{f}}}{A_f} \right| \neq 1$

Example

Consider the CP mirror processes:

$$B \rightarrow f \text{ and } \bar{B} \rightarrow \bar{f}$$

The CP asymmetry is defined as

$$A_{CP} \equiv \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})} \quad \rightarrow$$

The decay amplitudes are

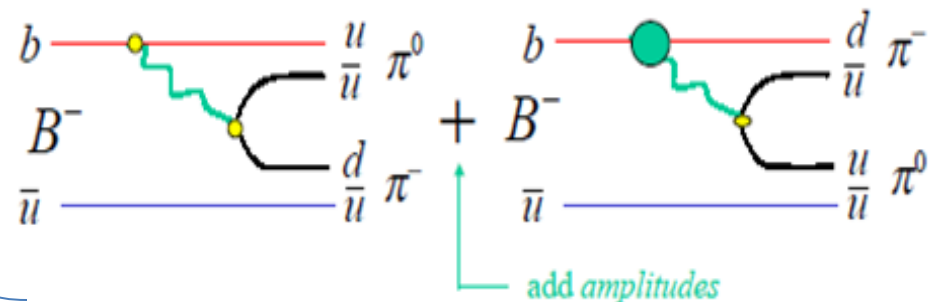
$$A_f = |A_f| e^{i\phi_{KM}} \text{ and } A_{\bar{f}} = |A_f| e^{-i\phi_{KM}}$$

Note that the KM phase changes sign.

However, $|A_f|^2 = |A_{\bar{f}}|^2 \Rightarrow \Gamma_f = \Gamma_{\bar{f}}$

We see no effect! This is so even though the weak interaction is in a sense maximally CP violating.

\Rightarrow **We need some sort of interference**, two amplitudes (i.e., two Feynman diagrams). Consider $B^- \rightarrow \pi^- \pi^0$



Direct CP violation

The resulting amplitudes are:

$$A_f = |A_1| e^{i\phi_{KM_1}} e^{i\phi_{S_1}} + |A_2| e^{i\phi_{KM_2}} e^{i\phi_{S_2}}$$

$$A_{\bar{f}} = |A_1| e^{-i\phi_{KM_1}} e^{i\phi_{S_1}} + |A_2| e^{-i\phi_{KM_2}} e^{i\phi_{S_2}}$$

Note that there is one more slight complication: the addition of a strong phase (but this is a *good* thing).

$$\Gamma_f = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\Delta\phi_{KM} \oplus \Delta\phi_S)$$

$$\Gamma_{\bar{f}} = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\Delta\phi_{KM} \ominus \Delta\phi_S)$$

Despite its conceptual and experimental “simplicity”, there are two problems with direct CP violation:

- Cases where there are two comparable amplitudes that are large are (probably) rare.
- The strong phases are poorly understood, making it difficult to extract the weak (KM) phases that are of greatest interest.

We need a better way. Such a way, which goes by the name of “Indirect CP Violation,” has been found and will be the topic of all that follows.

CP violation in mixing

This implies that the oscillation from meson to anti-meson is different from the oscillation from anti-meson to meson

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

Experimentally this is searched in semileptonic decays of B^0 and anti- B^0

$\Upsilon \rightarrow \bar{B}^0 B^0 \Rightarrow$ Production of B and anti-B

The B^0 meson decays to a positively charged lepton while anti- B^0 will decay to a negatively charged lepton



So, an event with two leptons with equal charge in the final state means that one of the two B- meson oscillated !

Comparison of oscillation rates

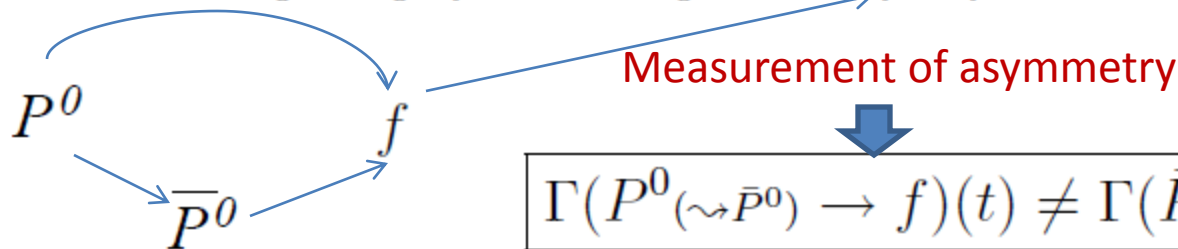


$$A_{CP} = \frac{N_{++} - N_{--}}{N_{++} + N_{--}} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2}$$

Mixing induced CP violation

→ CP violation in interference between a decay with and without mixing

An interesting category are CP-eigenstates, $f = \bar{f}$



$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}} = \frac{2C_f \cos \Delta mt - 2S_f \sin \Delta mt}{2 \cosh \frac{1}{2} \Delta \Gamma t + 2D_f \sinh \frac{1}{2} \Delta \Gamma t}$$

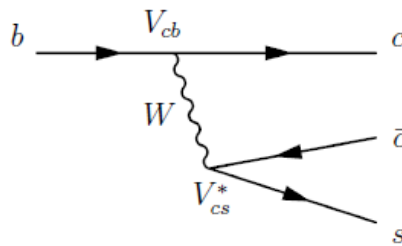
This simplifies considerably if the transition is dominated by only one amplitude, i.e. assuming that $|A_f| = |\bar{A}_f|$ (or $|\lambda_f| = 1$), so that $D_f = \Re \lambda_f$, $C_f = 0$ and $S_f = \Im \lambda_f$:

$$\rightarrow A_{CP}(t) = \frac{-\Im \lambda_f \sin \Delta mt}{\cosh \frac{1}{2} \Delta \Gamma t + \Re \lambda_f \sinh \frac{1}{2} \Delta \Gamma t}$$

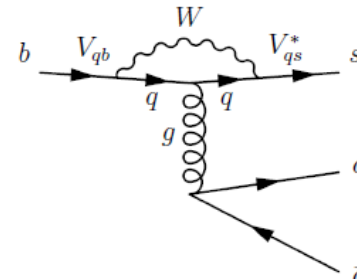
Angle of the Unitarity Triangle

β -measurement

The $b \rightarrow c\bar{c}s$ process



Tree



Penguin

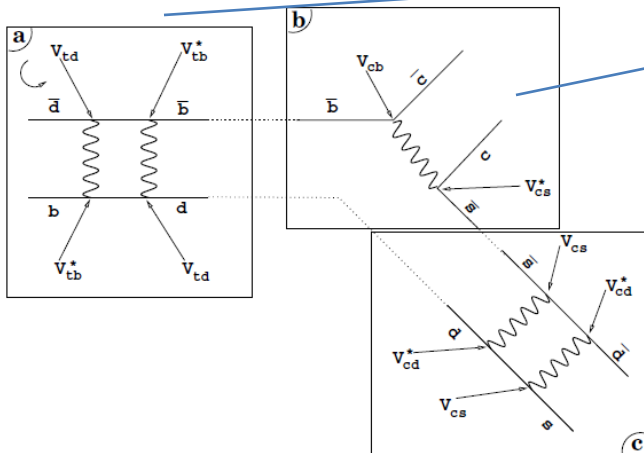
$$\eta_{J/\psi K_S} = -1$$

CP eigenstate

$$A_{CP}(t) = -\Im \lambda_f \sin(\Delta m t)$$

For an explicit, and important, example, consider the measurement of CP violation in $B \rightarrow J/\psi K_S^0$

$$\lambda_{J/\psi K_S^0} = \left(\frac{q}{p}\right)_{B^0} \left(\eta_{J/\psi K_S^0} \frac{\bar{A}_{J/\psi K_S^0}}{A_{J/\psi K_S^0}} \right) = - \left(\frac{q}{p}\right)_{B^0} \left(\frac{\bar{A}_{J/\psi \bar{K}^0}}{A_{J/\psi K^0}} \right) \left(\frac{p}{q}\right)_{K^0} \quad \left(\frac{q}{p}\right)_{B^0} = \sqrt{\frac{M_{12}}{M_{12}^*}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$$



$$\left(\frac{\bar{A}}{A}\right) = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}$$

$$\left(\frac{p}{q}\right)_K = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

$$\Im \lambda_{J/\psi K_S^0} = -\sin \left\{ \arg \left(\frac{V_{tb}^* V_{td} V_{cb} V_{cd}^*}{V_{tb} V_{td}^* V_{cb}^* V_{cd}} \right) \right\} = -\sin \left\{ 2 \arg \left(\frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right) \right\} \equiv \sin 2\beta$$

γ -measurement

➔ $\gamma \equiv \arg(\bar{\rho} + i\bar{\eta}) = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$

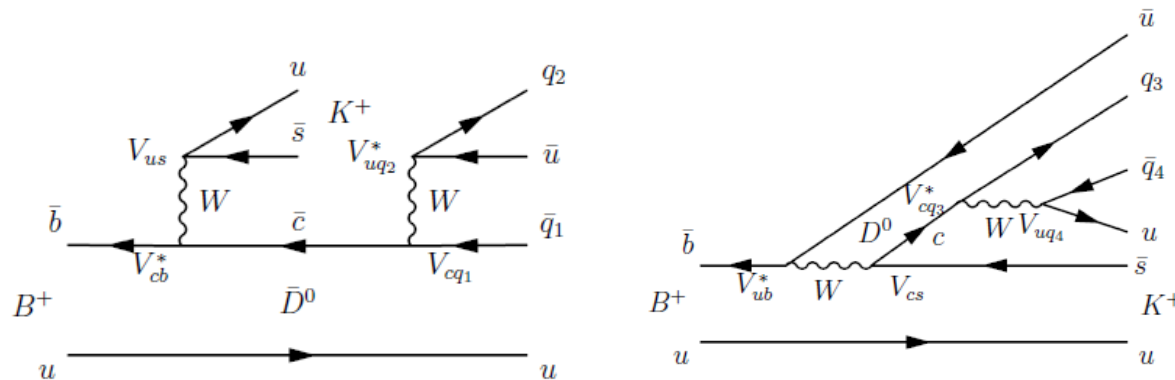
➔ γ is accessible in interference between various pairs of tree-level diagrams

Most poorly measured angle !!

Different approaches : interference between $b \rightarrow c\bar{u}s$ and $b \rightarrow \bar{c}us$ processes leading to the same final state

For example, consider the decays $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$

We define: $R_B \equiv \frac{\text{amplitude for } B^+ \rightarrow D^0 h}{\text{amplitude for } B^+ \rightarrow \bar{D}^0 h}$, $R_D \equiv \frac{\text{amplitude for } D^0 \rightarrow f}{\text{amplitude for } \bar{D}^0 \rightarrow f}$, $R = R_B R_D$



$$R = R_0 \frac{V_{ub}^* V_{cs}}{V_{cb}^* V_{us}} \frac{V_{cq3}^* V_{uq4}}{V_{cq1} V_{uq2}^*}$$

$$\arg(R) = \arg(R_0) + \gamma$$

γ -measurement example

GLW method

For $h = K^+$ and $f = K^+K^-$ we define $r_B \equiv \left| \frac{\text{amplitude for } B^+ \rightarrow D^0 K^+}{\text{amplitude for } B^+ \rightarrow \bar{D}^0 K^+} \right|$, $R_D = r_D e^{i\delta_D}$

Consider the CP eigenstates of the neutral D meson

$$D_+ = \frac{1}{\sqrt{2}}(D^0 + \bar{D}^0)$$
$$D_- = \frac{1}{\sqrt{2}}(D^0 - \bar{D}^0)$$

$$R_{\pm} \equiv 2 \frac{\Gamma(B^- \rightarrow D_{\pm} K^-) + \Gamma(B^+ \rightarrow D_{\pm} K^+)}{\Gamma(B^- \rightarrow D^0 K^-) + \Gamma(B^+ \rightarrow \bar{D}^0 K^+)} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma,$$

$$A_{\pm} \equiv \frac{\Gamma(B^- \rightarrow D_{\pm} K^-) - \Gamma(B^+ \rightarrow D_{\pm} K^+)}{\Gamma(B^- \rightarrow D_{\pm} K^-) + \Gamma(B^+ \rightarrow \bar{D}_{\pm} K^+)} = \frac{\pm 2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma}$$

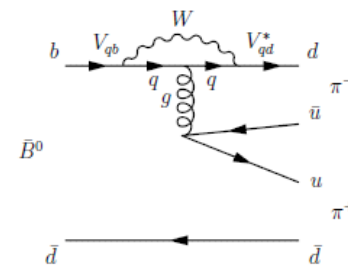
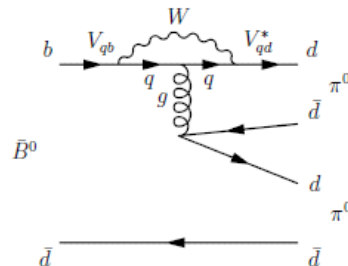
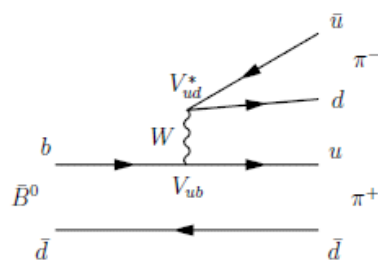
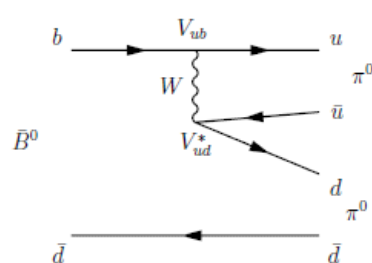
Thus, by measuring the two ratios and the two asymmetries we have four measurements which can be used to determine the three parameters r_B, δ_B , and γ . Unfortunately, there is an ambiguity under $(\delta_B, \gamma) \leftrightarrow (\pi - \delta_B, \pi - \gamma) \leftrightarrow (\pi + \delta_B, \pi + \gamma) \leftrightarrow (\gamma, \delta_B)$.

α -measurement

The angle α measures the phase of $V_{ud}V_{ub}^*$ relative to $V_{td}V_{tb}^*$

➡ time-dependent CP violation in the $b \rightarrow u\bar{u}d$ process

➡ The simplest channel to consider here is $B \rightarrow \pi\pi$ with tree-level and penguin diagrams



➡ $B^+ \rightarrow \pi^+\pi^0$ has no gluonic penguin contribution

➡ First, we note that the $\pi\pi$ system must be either in an $I = 0$ or $I = 2$ state

★ The tree-level process may have contributions from both isospins

★ the gluonic penguin amplitude can only have $I = 0$ contributions

The isospin relations are used to measure the angle α after the measurements of CP violating parameters in $B \rightarrow \pi^+\pi^-$

$\beta_s (\Phi_s)$ measurement

We may measure the angle β_s by considering $B_s \rightarrow J/\psi\phi$

$$\lambda_{J/\psi\phi} = (-)^\ell \frac{V_{ts} V_{tb}^* V_{cb} V_{cs}^*}{V_{ts}^* V_{tb} V_{cb}^* V_{cs}} = (-)^\ell e^{2i\beta_s}$$

Since the ϕ is also spin one, the orbital angular momentum can be $\ell = 0, 1, 2$.

➡ As with the measurement of β in $B^0 \rightarrow J/\psi K_S$, penguin pollution is expected to be small

Suppressed by $\sim |V_{us} V_{ub} / V_{cs} V_{cb}| \approx |\rho + i\eta| \lambda^2 \approx 0.02$

➡ maximum penguin effect of $\sim \pm 1^\circ$ on ϕ_s .

➡ In the B_s case, it turns out that $\Delta\Gamma_s \equiv \Delta\Gamma(B_s)$ cannot be neglected

$$\mathcal{A}_f(t) = \frac{\Gamma(\bar{B}_s^0(t) \rightarrow f) - \Gamma(B_s^0(t) \rightarrow f)}{\Gamma(\bar{B}_s^0(t) \rightarrow f) + \Gamma(B_s^0(t) \rightarrow f)} = \frac{S_f \sin \Delta m(B_s)t - C_f \cos \Delta m(B_s)t}{\cosh \frac{\Delta\Gamma_s t}{2} - \frac{2\Re\lambda}{1+|\lambda|^2} \sinh \frac{\Delta\Gamma_s t}{2}}$$

➡ Hence, both $\Delta\Gamma_s$ and ϕ_s are extracted in time-dependent fits to the decay distributions.

CP violation in K

The k_L and k_S states are not CP eigenstates since:

$$\mathbf{CP} |k_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (\mathbf{CP} |k_1\rangle + \epsilon \mathbf{CP} |k_2\rangle) = \frac{1}{\sqrt{1+|\epsilon|^2}} (|k_1\rangle - \epsilon |k_2\rangle) \neq |k_S\rangle$$

$$\mathbf{CP} |k_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (\mathbf{CP} |k_2\rangle + \epsilon \mathbf{CP} |k_1\rangle) = \frac{1}{\sqrt{1+|\epsilon|^2}} (-|k_2\rangle + \epsilon |k_1\rangle) \neq |k_L\rangle$$

$$\Gamma_{K^0 \rightarrow f}(t) = N (e^{-\Gamma_S t} + e^{-\Gamma_L t} |\eta_{+-}|^2 + 2e^{-\Gamma t} |\eta_{+-}| \cos(\Delta m t + \phi_{+-}))$$

$$\Gamma_{\bar{K}^0 \rightarrow f}(t) = N (e^{-\Gamma_S t} + e^{-\Gamma_L t} |\eta_{+-}|^2 - 2e^{-\Gamma t} |\eta_{+-}| \cos(\Delta m t + \phi_{+-}))$$

$$\eta_{+-} = \frac{1-\lambda_f}{1+\lambda_f}$$

➡ Let us consider the decays $K^0 \rightarrow \pi^+\pi^-$ and $\bar{K}^0 \rightarrow \pi^+\pi^-$ and define the parameter λ_f

$$\lambda_{\pi^+\pi^-} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{\pi^+\pi^-}}{A_{\pi^+\pi^-}} \quad \eta_{+-} \equiv \frac{\langle \pi^+\pi^- | T | K_L^0 \rangle}{\langle \pi^+\pi^- | T | K_S^0 \rangle} = \frac{p A_{\pi^+\pi^-} - q \bar{A}_{\pi^+\pi^-}}{p A_{\pi^+\pi^-} + q \bar{A}_{\pi^+\pi^-}} = \frac{1 - \lambda_{\pi^+\pi^-}}{1 + \lambda_{\pi^+\pi^-}}$$

If $\eta_{+-} \neq 0$ then that means $|\lambda_{\pi^+\pi^-}| \neq 1$ ➡ **CP violation in mixing:** ϵ

Similarly, for the decay to two neutral pions the parameter η_{00} is introduced

CP violation in decay: ϵ'

$$\eta_{+-} \neq \eta_{00} \Rightarrow$$

Implies that the decay contributes to the CP violation

$$\frac{\eta_{00}}{\eta_{+-}} \approx \frac{\epsilon - 2\epsilon'}{\epsilon + \epsilon'} \approx 1 - 3\frac{\epsilon'}{\epsilon} \Rightarrow$$

Measure of direct CP violation in kaon decay

The two pion system can occur in two distinct eigenstates of the strong interaction, namely $I = 0$ and $I = 2$. So we can decompose the two-pion states emanating from the K_L^0 and K_S^0 decay into the Isospin eigenstates:

$$|\pi^+\pi^-\rangle = \frac{1}{\sqrt{3}} \left(\sqrt{2}|2\pi, I=0\rangle + |2\pi, I=2\rangle \right) \quad |\pi^0\pi^0\rangle = \frac{1}{\sqrt{3}} \left(|2\pi, I=0\rangle - \sqrt{2}|2\pi, I=2\rangle \right)$$

$$\Rightarrow \langle 2\pi, I=0 | T | K^0 \rangle = \langle 2\pi, I=0 | T | \bar{K}^0 \rangle = A_0 e^{i\delta_0}$$

$$\begin{aligned} \langle 2\pi, I=2 | T | K^0 \rangle &= A_2 e^{i\delta_2} \\ \langle 2\pi, I=2 | T | \bar{K}^0 \rangle &= A_2^* e^{i\delta_2} \end{aligned}$$

$$\Rightarrow \eta_{+-} = \frac{\langle \pi^+\pi^- | H | K_L^0 \rangle}{\langle \pi^+\pi^- | H | K_S^0 \rangle} = \epsilon + \epsilon' (1 + \Delta)^{-1}$$

$$\Rightarrow \eta_{00} = \frac{\langle \pi^0\pi^0 | H | K_L^0 \rangle}{\langle \pi^0\pi^0 | H | K_S^0 \rangle} = \epsilon - 2\epsilon' (1 - 2\Delta)^{-1}$$

$$\left. \begin{aligned} F &= e^{i(\delta_2 - \delta_0)} \\ \Delta &= \frac{F}{\sqrt{2}} \frac{\Re A_2}{A_0} \\ \epsilon' &= \frac{iF}{\sqrt{2}} \frac{\Im A_2}{A_0} \end{aligned} \right\}$$

$$\left\{ \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = 1 - 6\Re \frac{\epsilon'}{\epsilon} \right\}$$

CP violation

Success of KM !

- Indirect CP violation in $K \rightarrow \pi\pi$ and $K \rightarrow \pi l \nu$ is given by

$$|\epsilon_k| = (2.229 \pm 0.012) \times 10^{-3}$$

- Direct CP violation in $K \rightarrow \pi\pi$ decays is given by

$$\epsilon'/\epsilon = (1.65 \pm 0.26) \times 10^{-3}$$

- CP violation in the interference of mixing and decay in $B \rightarrow J/\psi K_s$ and other, related modes is given by

$$S_{J/\psi K_s} = 0.681 \pm 0.025$$

Other measurements : $S_{K^+K^-K_s}$, $S_{D^{*+}D^{*-}}$, $S_{\eta'K_s}$, $S_{f_0K_s}$,

$A_{K^\mp K^\pm}$ \Rightarrow All these measurements are consistent with KM mechanism.

B-Physics: Goal

Quark Mixing:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM Phenomenology:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

Wolfenstein Parametrization:

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

- Consistency check in the SM !!
- Searches for NP evidences !!

Unitarity Triangle

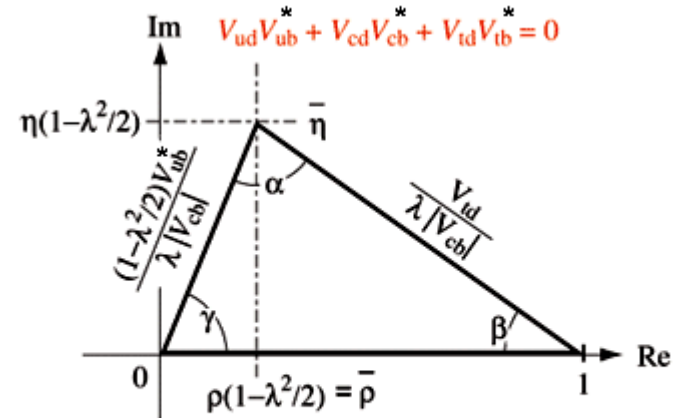


Figure 1. The unitarity relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ drawn in the complex $[\bar{\rho}, \bar{\eta}]$ plane.

Jarlskog's measure of CP violation :
 $2 \times \text{Area of the UT}$

Construction: UT

- The length of the sides of the UT :

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$

$$R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1 - \bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|.$$

- The angle β and γ :

$$V_{td} = |V_{td}|e^{-i\beta}, \quad V_{ub} = |V_{ub}|e^{-i\gamma}$$

- The Unitarity relation :

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$

- The angle α can be obtained :

$$\alpha + \beta + \gamma = 180^\circ$$

- Combining all these :

$$\bar{\eta} = \pm \sqrt{R_b^2 - \bar{\varrho}^2}, \quad \bar{\varrho} = \frac{1 + R_b^2 - R_t^2}{2}$$

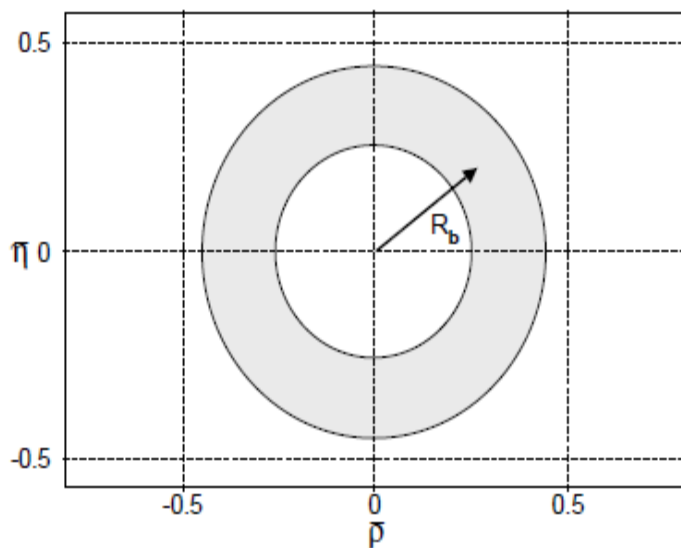
Role of $|V_{ub}|$ and $|V_{cb}|$

- ✓ $|V_{ub}|, |V_{cb}|$ hence R_b are determined from tree level decays !



Expected to be free of NP effects !!

- ✓ They are universal fundamental constants valid in any extension of the SM!!



This tells us that the apex of the unitarity triangle lies in the band shown

To find where the apex lies on the UT we have to look at other decays !!

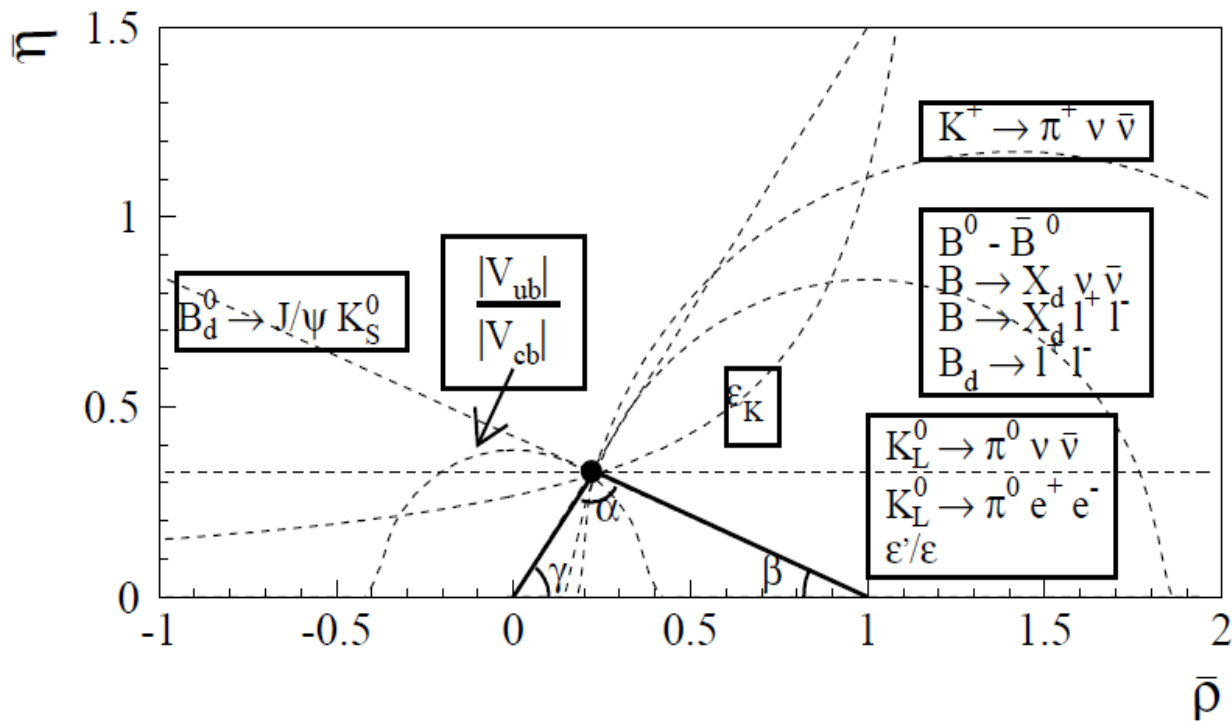
Most promising in this respect are the so-called loop induced decays and CP violating B-decays !!

M. Battaglia et al. [arXiv:hep-ph/0304132v2](#)

- ✓ **Precise determination of $|V_{ub}|, |V_{cb}|$ is of utmost importance !**

Ideal UT

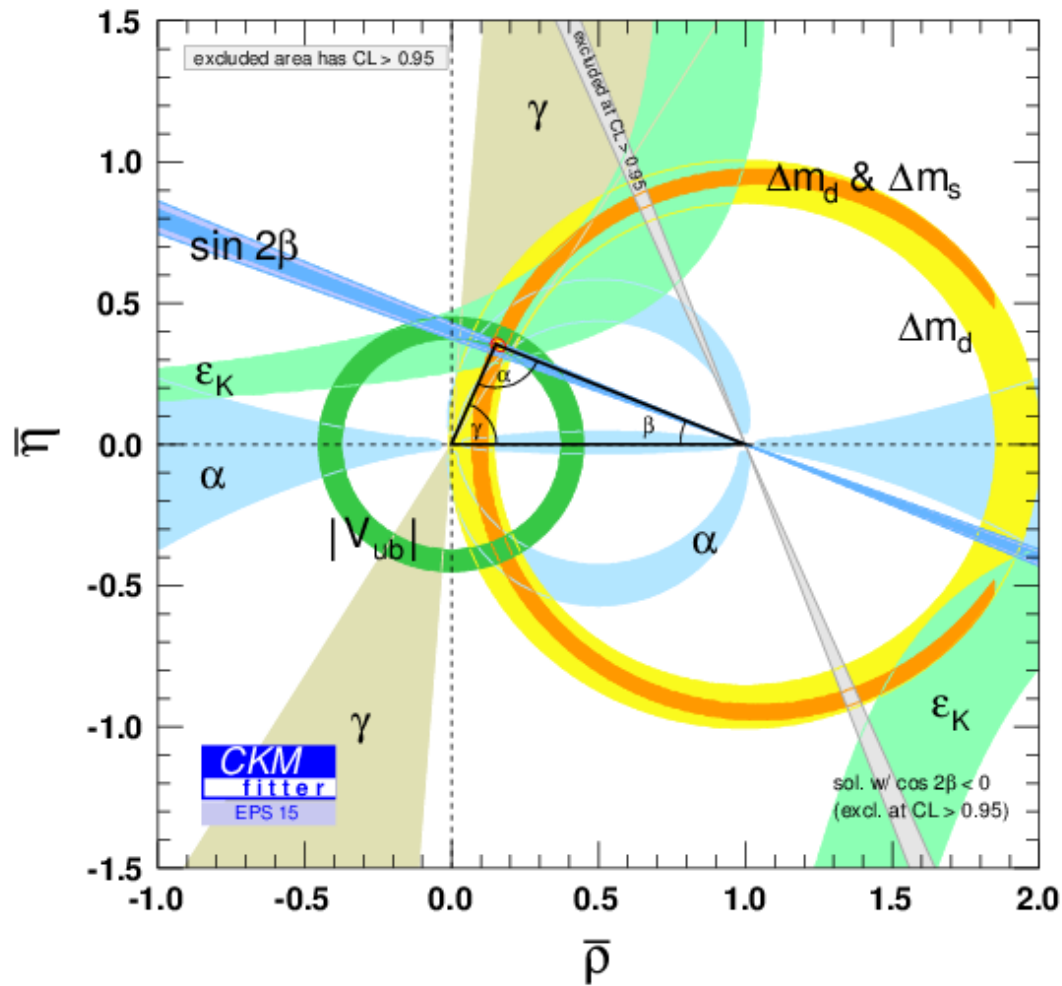
- ✓ Various curves in the (ρ, η) plane extracted from different decays and transitions using the SM formulae cross each other at a single point
- ✓ The angles (α, β, γ) in the resulting triangle agree with those extracted from CP asymmetries.



M. Battaglia et al.
[arXiv:hep-ph/0304132v2](https://arxiv.org/abs/hep-ph/0304132v2)

- ✓ Any inconsistencies in the (ρ, η) plane will then give us some hints about the physics beyond the SM !!

UT Fit Results

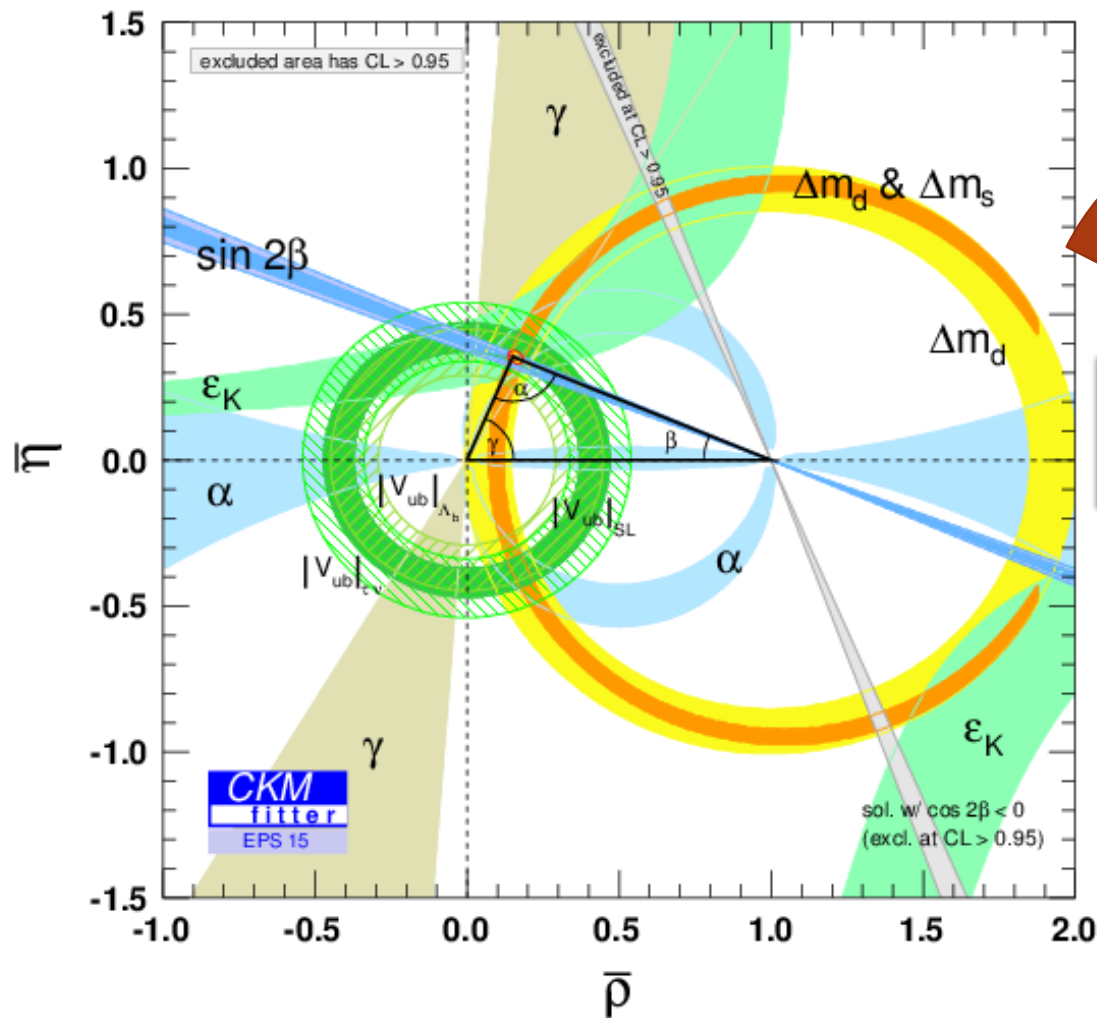


$$A = 0.810^{+0.018}_{-0.024}, \quad \lambda = 0.22548^{+0.00068}_{-0.00034},$$

$$\bar{\rho} = 0.145^{+0.013}_{-0.007}, \quad \bar{\eta} = 0.343^{+0.011}_{-0.012}.$$

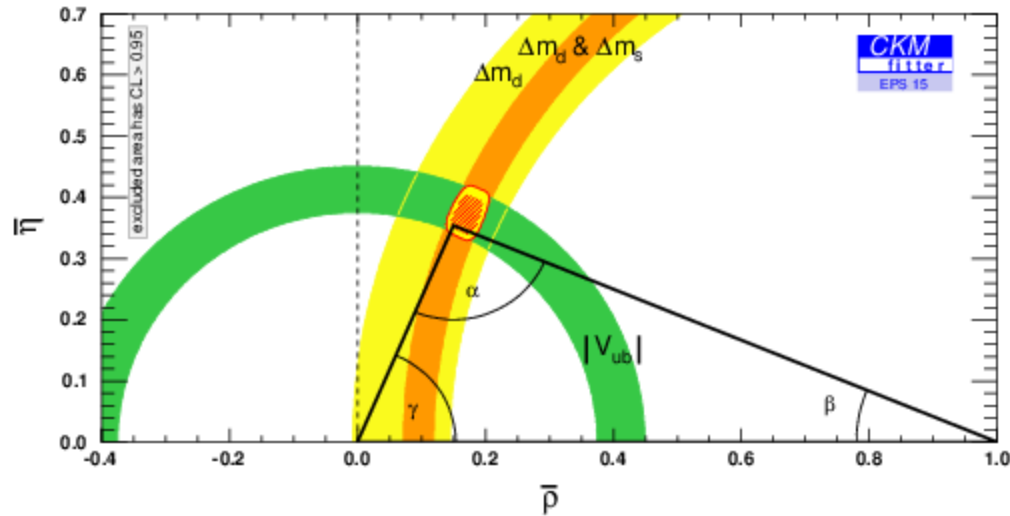
Exist a unique preferred region
defined by the entire set of
obseables under consideration.

UT fit

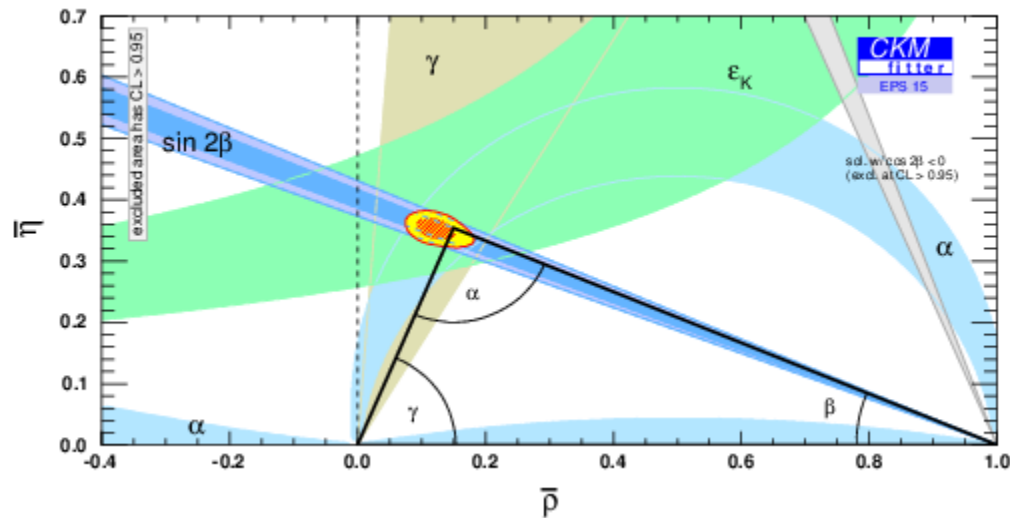


Considering various measurements
of $|V_{ub}|$

Consistency of UT fit

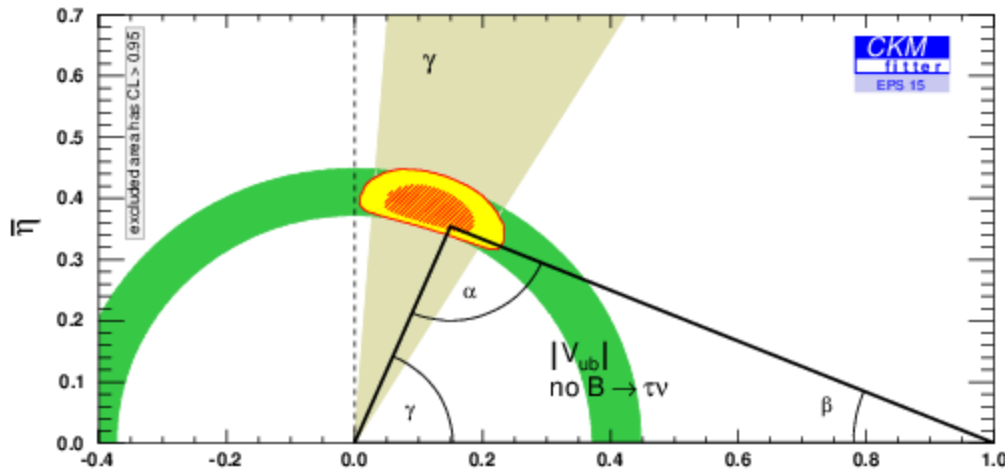


CP conserving observables

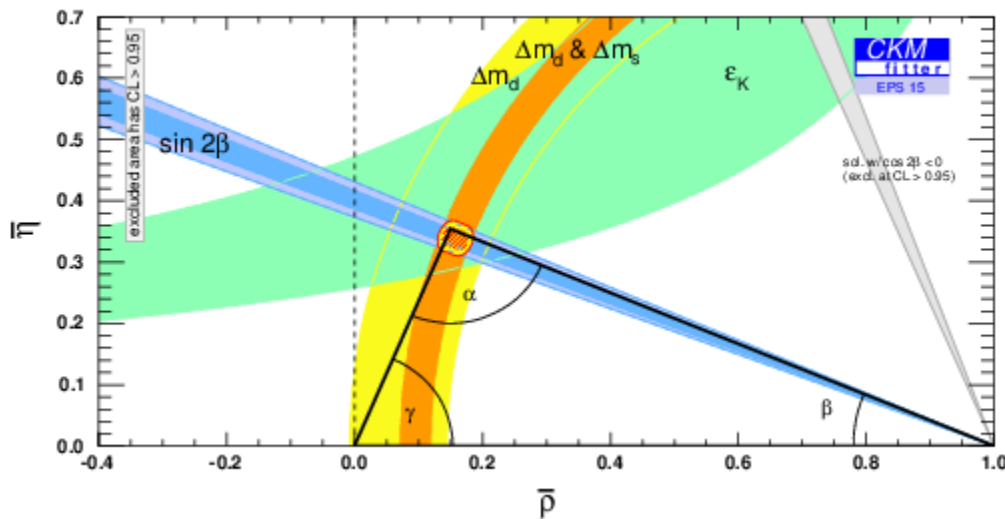


CP violating observables

Consistency of UT fit : Tree vs Loop



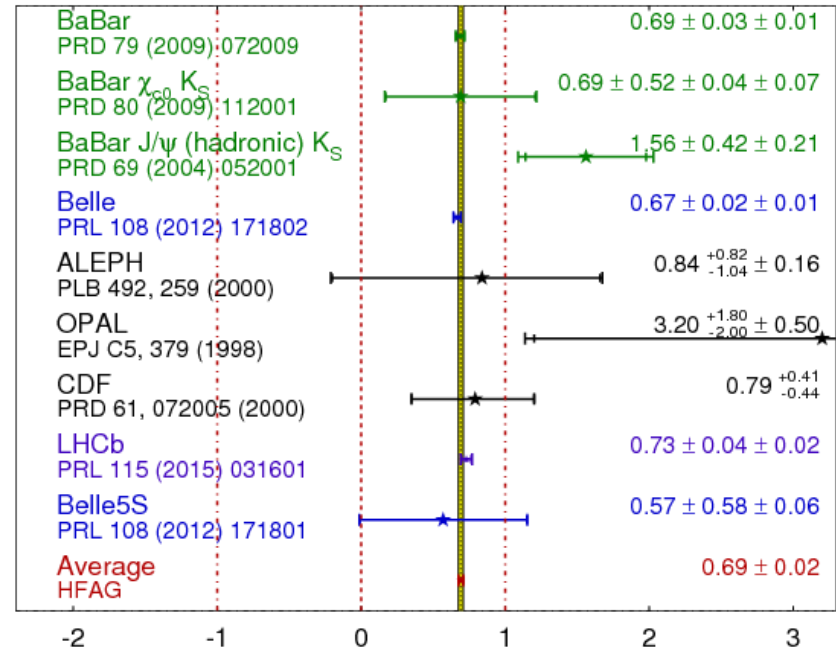
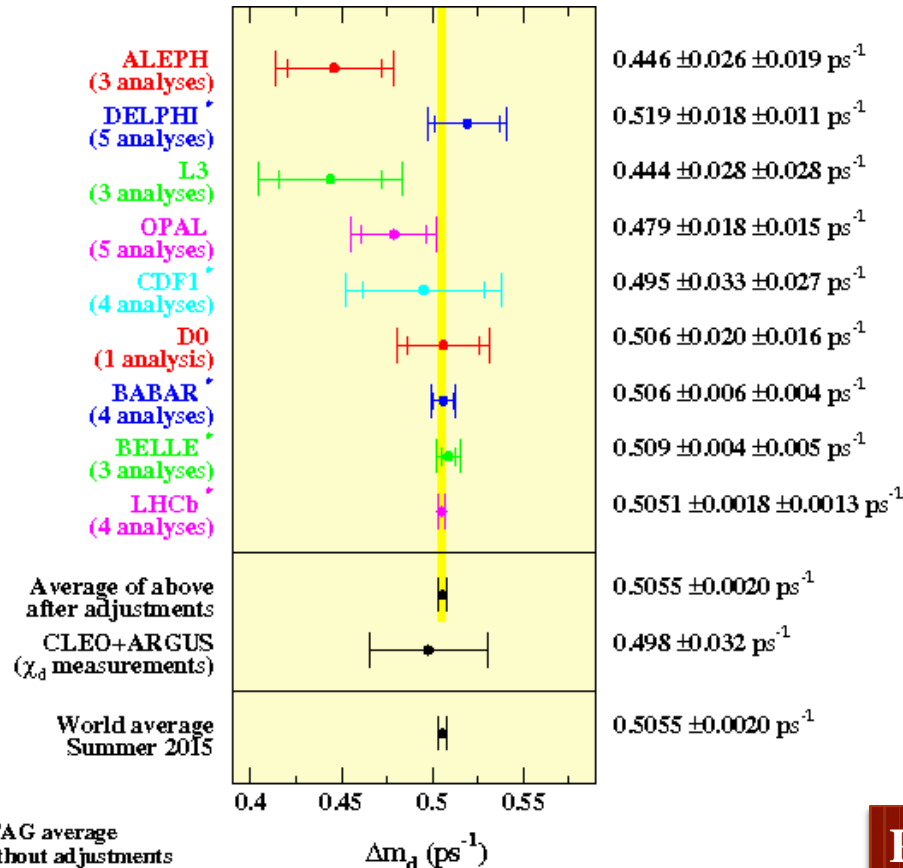
Only Tree level processes



Loop level processes

Δm_d & $\sin(2\beta)$ (Exp.)

$\sin(2\beta) \equiv \sin(2\phi_1)$ **HFAG**
Moriond 2015
PRELIMINARY

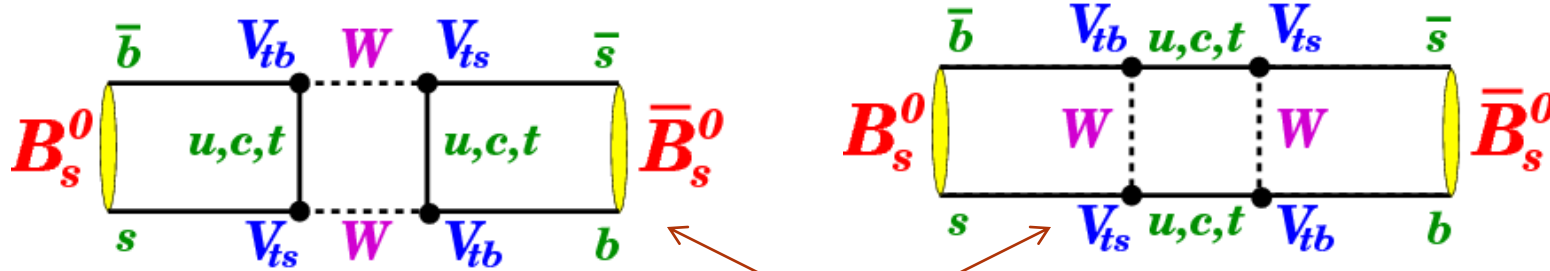


Prediction

$\Rightarrow \sin 2\beta = 0.771^{+0.017}_{-0.041}$

$\Delta M_d^{\text{SM}} = 0.543 \pm 0.091 \text{ ps}^{-1}$

CP asymmetry in B_s



$$\Delta M_s := M_H^s - M_L^s$$

$$= 2 |M_{12}^s| \left(1 - \frac{|\Gamma_{12}^s|^2 \sin^2 \phi_{12}^s}{8 |M_{12}^s|^2} + \dots \right)$$

$$\Delta \Gamma_s := \Gamma_L^s - \Gamma_H^s$$

$$= 2 |\Gamma_{12}^s| \cos \phi_{12}^s \left(1 + \frac{|\Gamma_{12}^s|^2 \sin^2 \phi_{12}^s}{8 |M_{12}^s|^2} + \dots \right)$$

$$\phi_{12}^s := \arg \left(-\frac{M_{12}^s}{\Gamma_{12}^s} \right) = \pi + \phi_M - \phi_\Gamma$$

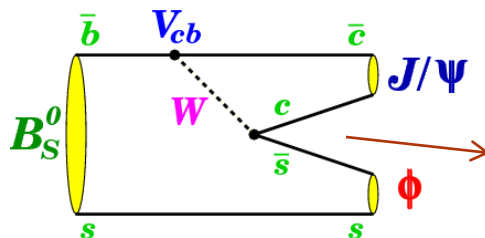
Due to weak interaction B_s can transform to anti- B_s and vice versa .

Dispersive part

Absorptive part

$$M_{12}^s = |M_{12}^s| e^{i\phi_M}$$

$$\Gamma_{12}^s = |\Gamma_{12}^s| e^{i\phi_\Gamma} . \quad e^{i\phi_M} = \frac{V_{ts}^* V_{tb}}{V_{ts} V_{tb}^*}$$



$$\phi_s = -\arg(\lambda_f) = -\arg \left(\frac{q}{p} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \right)$$

$$= -\pi + \phi_M - \arg \left(\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \right) .$$

There can also be new physics contributions to Γ_{12}^s , e.g. by modified tree-level operators or by new $b s \tau \tau$ -operators, as discussed below.

Dighe, Kundu, SN, Bauer, Haisch, Bobeth ...

Other CP Obs.

Time dependent CP asymmetries in $B_s \rightarrow f$

Large

$$A_{CP,f}(t) = - \frac{\mathcal{A}_{CP}^{\text{dir}} \cos(\Delta M_s t) + \mathcal{A}_{CP}^{\text{mix}} \sin(\Delta M_s t)}{\cosh(\frac{\Delta \Gamma_s t}{2}) + \mathcal{A}_{\Delta \Gamma} \sinh(\frac{\Delta \Gamma_s t}{2})}$$

$$\mathcal{A}_{\Delta \Gamma} = - \frac{2|\lambda_f|}{1 + |\lambda_f|^2} \cos[\arg(\lambda_f)] = - \frac{2|\lambda_f|}{1 + |\lambda_f|^2} \cos[\phi_s]$$

Hard to measure in B_d decays since $\Delta \Gamma_d$ is expected to be small !

Semileptonic CP asymmetries in B_d & B_s

$$a_{sl}^q \equiv a_{fs}^q = \frac{\Gamma(\overline{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \overline{f})}{\Gamma(\overline{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \overline{f})} = \left| \frac{\Gamma_{12}^q}{M_{12}^q} \right| \sin \phi_q$$

Di-muon asymmetry

C_d and C_s are roughly equal !

$$A_{CP} = C_d a_{sl}^d + C_s a_{sl}^s + \frac{1}{2} C_{\Delta \Gamma_d} \frac{\Delta \Gamma_d}{\Gamma_d}$$

SM predictions

Mass and Life time differences between heavy light mass eigenstates

$$\Delta M_s^{\text{SM},2015} = (18.3 \pm 2.7) \text{ ps}^{-1}$$

$$\Delta M_d^{\text{SM},2015} = (0.528 \pm 0.078) \text{ ps}^{-1}$$

$$\Delta \Gamma_s^{\text{SM},2015} = (0.088 \pm 0.020) \text{ ps}^{-1}$$

$$\Delta \Gamma_d^{\text{SM},2015} = (2.61 \pm 0.59) \cdot 10^{-3} \text{ ps}^{-1}$$

Semileptonic CP asymmetris

$$\begin{aligned} a_{\text{fs}}^{d,\text{SM},2015} &= (-4.7 \pm 0.6) \cdot 10^{-4} , \\ \phi_{12}^{d,\text{SM},2015} &= (-0.096 \pm 0.025) \text{ rad} \\ &= -5.5^\circ \pm 1.4^\circ . \end{aligned}$$

$$\begin{aligned} a_{\text{fs}}^{s,\text{SM},2015} &= (2.22 \pm 0.27) \cdot 10^{-5} \\ \phi_{12}^{s,\text{SM},2015} &= (4.6 \pm 1.2) \cdot 10^{-3} \text{ rad} \\ &= 0.26^\circ \pm 0.07^\circ . \end{aligned}$$

Measured Semileptonic asymmetries

Measured asymmetries in the decay $B_s \rightarrow D_s \mu X$

$$a_{sl}^{s D^0} = -1.12 \pm 0.74 \pm 0.17\%,$$

$$a_{sl}^{s LHCb} = -0.06 \pm 0.50 \pm 0.36\%$$

$$a_{sl}^{s HFAG} = -0.48 \pm 0.48\%.$$

$$a = \frac{n^+ - n^-}{n^+ + n^-},$$

$$A = \frac{N^{++} - N^{--}}{N^{++} + N^{--}}$$

Includes the measurements of simultaneous study of the inclusive semileptonic single charge asymmetry and like sign di-muon charge asymmetry by DØ

Experiment	measured a_{sl}^d (%)
LHCb $D^{(*)}\mu\nu X$	$-0.02 \pm 0.19 \pm 0.30$
D0 $D^{(*)}\mu\nu X$	$+0.68 \pm 0.45 \pm 0.14$
BaBar $D^*\ell\nu X$	$+0.29 \pm 0.84^{+1.88}_{-1.61}$
BaBar $\ell\ell$	$-0.39 \pm 0.35 \pm 0.19$

Average of all the measurements

$$-0.0015 \pm 0.0017$$

FCNC

FCNC : Loop process

GIM : Kaon system

- Cabibbo fixed one issue ($s \rightarrow u$ transition), but introduced another one
- If doublet of weak interaction is (u, d') , then also Z^0 can couple to $d'\bar{d}'$

- What does it mean in terms of original quarks?

$$u\bar{u} + d\bar{d}\cos^2\theta + s\bar{s}\sin^2\theta + (s\bar{d} + \bar{s}d)\sin\theta\cos\theta$$

The last term would allow FCNC at tree level !

- To existing doublet $(u, d') = (u, d\cos(\theta) + s\sin(\theta))$ add second one
 $(c, s') = (c, d\cos(\theta) - s\sin(\theta))$

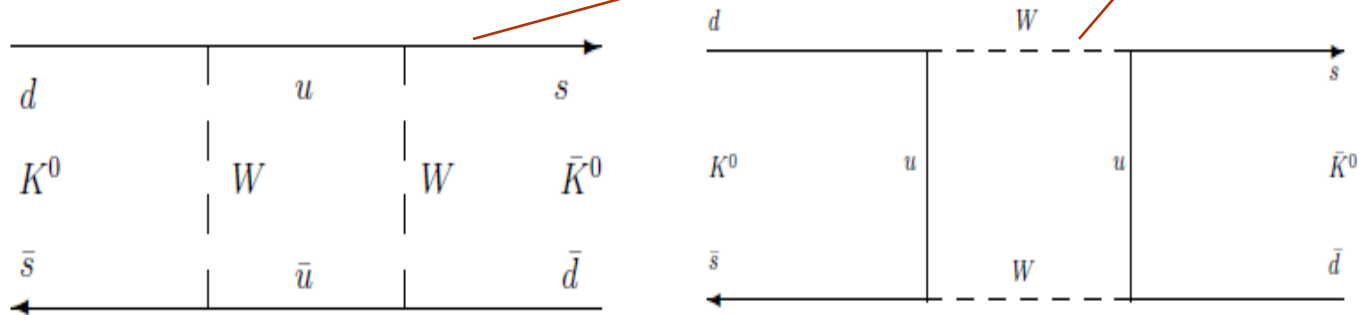
← Glashow, Iliopoulos, Maiani :

Mass difference

$$K_L^0 - K_S^0$$

- $K_S^0 = (d\bar{s} + s\bar{d})/\sqrt{2}$, $K_L^0 = (d\bar{s} - s\bar{d})/\sqrt{2}$
- $\tau_S \sim 10^{-10} \text{s}$; $\tau_L \sim 10^{-8} \text{s}$
- K_S^0 decays into two pions, K_L^0 decay into three pions
- $\Delta m = m_L - m_S \sim 10^{-12} \text{ MeV (exp)}$

$$\Delta m \sim 10^{-8} \text{ MeV}$$



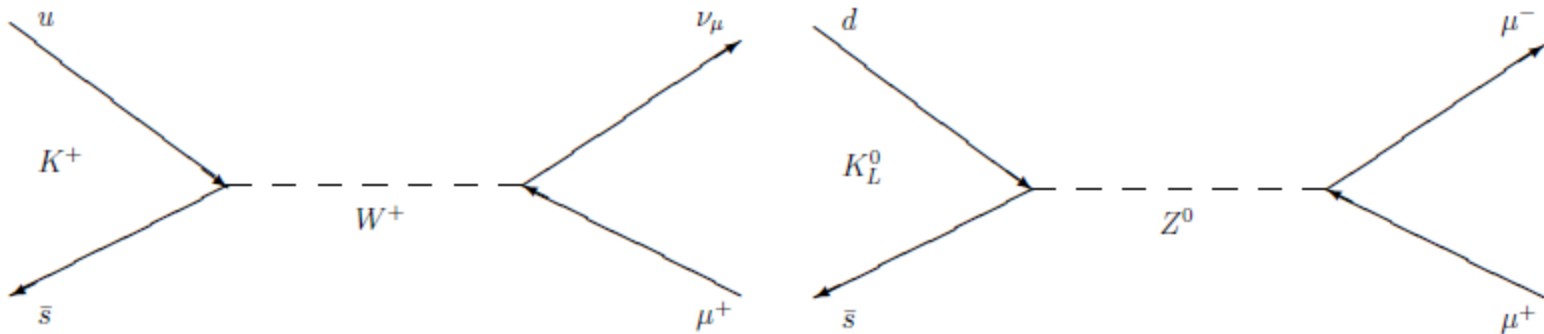
- including higher order terms leads to divergencies

Inclusion the additional diagrams with charm will explain the discrepancy !!

Rare decays

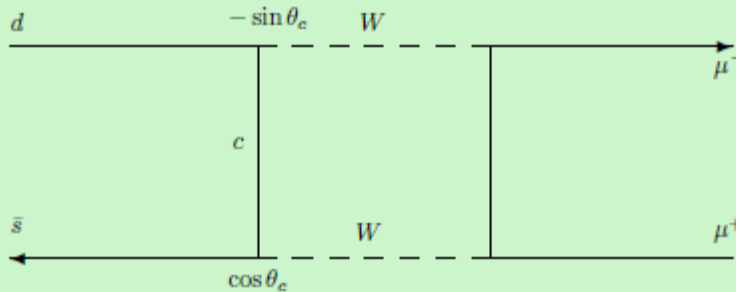
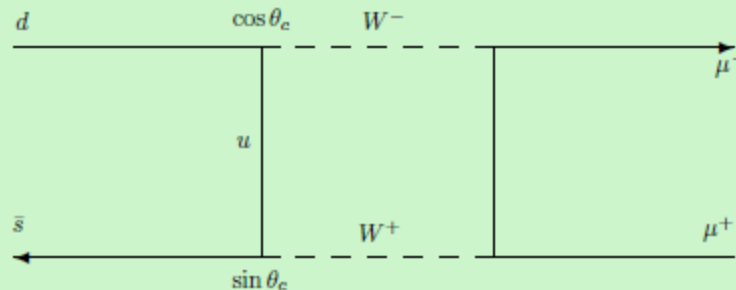
Neutral current

- strangeness changing neutral weak interactions do not occur
- neutral decay $K_L^0 \rightarrow \mu^+ \mu^-$ branching ratio only 9×10^{-9}
- analogous decay $K^+ \rightarrow \mu^+ \nu_\mu$ is fully allowed



Rare decays

even if first order amplitude is zero there is the second order contribution



if all of the quark masses were degenerate then there would be no strangeness changing neutral current effect in any order since these two graphs would cancel exactly

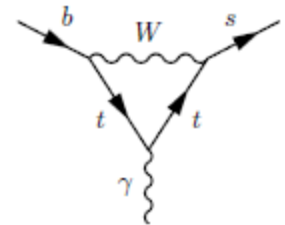
$b \rightarrow s$ decays

B- > X_s γ: Motivation

- The decay width $\Gamma(\text{B} \rightarrow \text{X}_s \gamma)$ is well approximated by $\Gamma(\text{b} \rightarrow \text{s} \gamma)$:

$$\Gamma(\bar{B} \rightarrow X_s \gamma) = \Gamma(b \rightarrow s \gamma) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right) \quad , \quad \begin{array}{l} E_0 \sim m_b/2 \\ (m_b - 2E_0 \gg \Lambda_{\text{QCD}}) \end{array}$$

- $\Gamma(\text{b} \rightarrow \text{s} \gamma)$ can be analyzed in perturbation theory !
- $\text{b} \rightarrow \text{s} \gamma$ is a *flavor changing neutral current (FCNC) process* !
- In SM no FCNC at tree level \longrightarrow only at the loop level !
- The relevant operator $\longrightarrow Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b$
- $\text{B} \rightarrow \text{X}_s \gamma$ is an important probe of new physics (NP) !



Experimental world average (HFAG, 2.08.2012):

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{EXP}} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

B- \rightarrow X_s γ : SM

➤ The effective Hamiltonian : $\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$

➤ At leading order only $Q_{7\gamma}$ - Q_{8g} contribute

➤ At higher order contribution from Q_i - Q_j are also important ..

✓ Most Important contributions are from ,

$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b \\ Q_1^q &= (\bar{q}b)_{V-A} (\bar{s}q)_{V-A} \quad (q = u, c) \end{aligned}$$

$$\Gamma(b \rightarrow X_s \gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{em}}{32 \pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^8 C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

Wilson Coefficients $C_i(\mu_b)$ are known at NNLO \Rightarrow

$|C_{1,2}(\mu_b)| \sim 1$, $|C_{3,4,5,6}(\mu_b)| < 0.07$, $|C_7(\mu_b)| \sim -0.3$, $|C_8(\mu_b)| \sim -0.15$

$G_{ij}(E_0, \mu_b) \Rightarrow$ Matrix elements of O_1, \dots, O_8

$\text{Br}(\text{B}^- \rightarrow X_s \gamma)$

SM estimate [hep-ph/0609232]:

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4} \quad \text{Misiak, 2015}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, 3% from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, 3% parametric

$$(3.36 \pm 0.23) \times 10^{-4}$$

SM prediction after adding the new updates on NNLO and power corrections

Experimental world average (HFAG, 2.08.2012):

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{EXP}} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

Experiment agrees with the SM at better than $\sim 1\sigma$ level.

Uncertainties: TH $\sim 7\%$, EXP $\sim 6.5\%$.

CP asymmetry in $b \rightarrow s\gamma$

CP asymmetry in $B \rightarrow X_s \gamma$ decays sensitive to NP !

$$\mathcal{A}_{CP}(b \rightarrow s\gamma) = \frac{\Gamma(\bar{B} \rightarrow \bar{X}_s \gamma) - \Gamma(B \rightarrow X_s \gamma)}{\Gamma(\bar{B} \rightarrow \bar{X}_s \gamma) + \Gamma(B \rightarrow X_s \gamma)}$$

SM prediction

$$-0.6\% < \mathcal{A}_{CP}(B \rightarrow X_s \gamma)_{\text{SM}} < 2.8\%$$

Measured value

$$\mathcal{A}_{CP}^{B \rightarrow X_s \gamma} = (1.7 \pm 1.9 \pm 1.0)\%$$

Consistent with each other

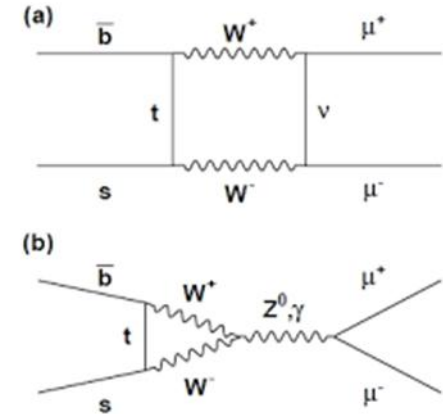
$$\Delta \mathcal{A}_{CP} = B^+ \rightarrow X_s \gamma - B^0 \rightarrow X_s \gamma \longrightarrow \text{Sensitive to } \Im \left(\frac{C_8}{C_7} \right)$$

Can be constrained using a precise measurement of the CP asymmetry difference.

Rare decays: $B_s \rightarrow \mu\mu$

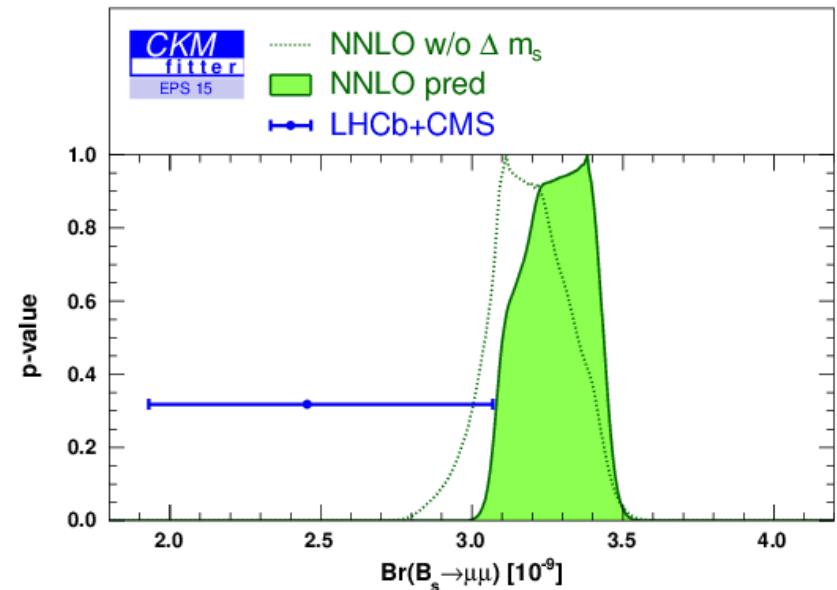
SM branching fraction

$$\frac{m_{B_q} m_\ell^2}{8\pi} \sqrt{1 - \frac{4m_\ell^2}{m_{B_q}^2}} \left(\frac{G_F m_W}{\pi} \right)^4 |V_{tb} V_{tq}^*|^2 |C_{10}(\mu, x_t)|^2 \frac{f_{B_q}^2}{\Gamma_H^q}$$



SM predictions

$$\begin{aligned} \overline{\mathcal{B}}(B_s \rightarrow \bar{e}e) &= (8.54 \pm 0.55) \times 10^{-14}, \\ \overline{\mathcal{B}}(B_s \rightarrow \bar{\mu}\mu) &= (3.65 \pm 0.23) \times 10^{-9}, \\ \overline{\mathcal{B}}(B_s \rightarrow \bar{\tau}\tau) &= (7.73 \pm 0.49) \times 10^{-7}, \\ \overline{\mathcal{B}}(B_d \rightarrow \bar{e}e) &= (2.48 \pm 0.21) \times 10^{-15}, \\ \overline{\mathcal{B}}(B_d \rightarrow \bar{\mu}\mu) &= (1.06 \pm 0.09) \times 10^{-10}, \\ \overline{\mathcal{B}}(B_d \rightarrow \bar{\tau}\tau) &= (2.22 \pm 0.19) \times 10^{-8}, \end{aligned}$$



Why NP ?

- History of matter and antimatter in the Universe can not be accounted by SM CP violation.
- Strong CP problem -> CP violation in Strong interaction is very small.
- Dark matter/energy puzzle.

Conclusions

- Heavy flavour studies are of fundamental importance
- Their lessons can not be obtained any other way
- None of the novel successes of SM weaken the case for NP ---TeV scale NP !
- The CP studies “instrumentalized” to analyze the NP